

Practice Problems

1. Differentiate each of the following.

a) $f(x) = \sqrt[3]{x^2} - \frac{3}{x^6} + e^2$

h) $p(y) = \frac{2}{y^3 + y}$

n) $f(x) = \cot x$

b) $g(x) = \sin^2 x + \cos^2 x$

i) $f(x) = \log_3(2x - 1) + \frac{1}{\ln x}$

o) $g(x) = \sqrt{\cos^2 x + 2}$

c) $f(x) = \ln(x^2 + 1)$

j) $f(x) = \csc(3x - 1)$

p) $f(y) = \tan(y^2 - 1)$

d) $f(a) = (a^3 - 1) \sin 2a$

k) $v(t) = \frac{t^2}{\sin^2 2t}$

q) $r(\theta) = -2 \csc^2(5\theta)$

e) $m(x) = \sqrt[5]{4x^6 + x^2 + 3}$

l) $r(t) = \ln(\tan t)$

r) $f(x) = \sin(10x) \cos(10x)$

f) $v(t) = \ln(\sqrt[3]{t^4 + t^2 + 1})$

m) $f(m) = \frac{2m - 3}{m + 5}$

s) $f(x) = \sqrt[7]{x^4} \cos x + 5$

g) $t(\alpha) = \cos(\alpha^2 + 1)$

t) $f(x) = \frac{2x^8 - 5}{\cos x}$

2. Differentiate each of the following.

a) $f(x) = \sin^{-1} 3x$

b) $g(x) = \tan^{-1}(x^5)$

c) $h(r) = \cos^{-1} r$

3. Compute each of the following integrals.

a) $\int (-5w + 1)^3 dw$

d) $\int_0^{\pi/4} \sin 2\theta d\theta$

g) $\int_1^6 \frac{1}{x^2} dx$

b) $\int_0^2 3t^2 - 6t + 1 dt$

e) $\int 2 - u + \sin 2u du$

h) $\int \sqrt[3]{x^2} + \cos 5x dx$

c) $\int_1^2 \frac{1}{x} dx$

f) $\int \frac{1}{\sqrt{1 - x^2}} dx$

4. Compute each of the following integrals.

a) $\int \frac{1}{1 + x^2} dx$

b) $\int \frac{x^2}{1 + x^2} dx$

c) $\int_{1/2}^1 \frac{1}{\sqrt{1 - x^2}} dx$

Practice Problems - Answers

$$1.) \quad \text{a) } f'(x) = \frac{2}{3\sqrt[3]{x}} + \frac{18}{x^7} = \frac{2\sqrt[3]{x^2}}{3x} + \frac{18}{x^7} \quad \text{b) } g'(x) = 0 \quad \text{c) } f'(x) = \frac{2x}{x^2 + 1}$$

$$\text{d) } f'(a) = 3a^2 \sin 2a + 2(a^3 - 1)(\cos 2a) \quad \text{e) } m'(x) = \frac{1}{5} \frac{24x^5 + 2x}{\sqrt[5]{(4x^6 + x^2 + 3)^4}} = \frac{1}{5} \frac{24x^5 + 2x}{4x^6 + x^2 + 3} \sqrt[5]{4x^6 + x^2 + 3}$$

$$\text{f) } v'(t) = \frac{4t^3 + 2t}{3(t^4 + t^2 + 1)} \quad \text{g) } t'(\alpha) = -2\alpha \sin(\alpha^2 + 1) \quad \text{h) } p'(y) = -\frac{2(3y^2 + 1)}{(y^3 + y)^2} = -\frac{6y^2 + 2}{(y + y^3)^2}$$

$$\text{i) } f'(x) = \frac{2}{(\ln 3)(2x - 1)} - \frac{1}{x \ln^2 x} \quad \text{j) } f'(x) = -3 \csc(3x - 1) \cot(3x - 1)$$

$$\text{k) } v'(t) = \frac{2t(\sin 2t - 2t \cos 2t)}{\sin^3 2t} \quad \text{l) } r'(t) = \tan t + \cot t \quad \text{m) } f'(m) = \frac{13}{(m + 5)^2}$$

$$\text{n) } f'(x) = -\csc^2 x = -\cot^2 x - 1 \quad \text{o) } g'(x) = -\frac{\sin x \cos x}{\sqrt{\cos^2 x + 2}} \quad \text{p) } f'(y) = 2y(\tan^2(y^2 - 1) + 1)$$

$$\text{q) } r'(\theta) = 20 \frac{\cos 5\theta}{\sin^3 5\theta} = 20 \csc^2 5\theta \cot 5\theta \quad \text{r) } f'(x) = 10 \cos(20x)$$

$$\text{s) } f'(x) = \frac{\sqrt[7]{x^4}}{x} \cos x - \sqrt[7]{x^4} \sin x \quad \text{t) } f'(x) = \frac{16x^7 \cos x + (2x^8 - 5) \sin x}{\cos^2 x}$$

$$2.) \quad \text{a) } f'(x) = \frac{3}{\sqrt{1 - 9x^2}} \quad \text{b) } g'(x) = \frac{5x^4}{x^{10} + 1} \quad \text{c) } h'(r) = -\frac{1}{\sqrt{1 - r^2}}$$

$$3.) \quad \text{a) } -\frac{1}{20}(-5w + 1)^4 + C \quad \text{b) } -2 \quad \text{c) } \ln 2 \quad \text{d) } \frac{1}{2} \quad \text{e) } 2u - \frac{1}{2}u^2 - \frac{1}{2}\cos 2u + C$$

$$\text{f) } \arcsin x + C \quad \text{g) } \frac{5}{6} \quad \text{h) } \frac{3}{5}x^{5/3} + \frac{1}{5}\sin 5x + C$$

$$4.) \quad \text{a) } \tan^{-1} x + C \quad \text{b) } x - \tan^{-1} x + C \quad \text{c) } \frac{\pi}{3}$$