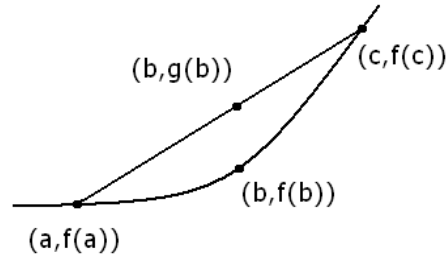


Suppose that f is a function that is concave up on a open interval I . To show: f is continuous on I .

Claim 1. Let $a < b < c$ be numbers in the interval I . Let m_{ab} be the slope of the line segment defined by the points $(a, f(a))$ and $(b, f(b))$; let m_{ac} be the slope of the line segment defined by the points $A(a, f(a))$ and $C(c, f(c))$; let m_{bc} be the slope of the line segment defined by the points $B(b, f(b))$ and $C(c, f(c))$. Then $m_{ab} \leq m_{ac} \leq m_{bc}$.

proof. Consider the picture shown below. Let $g(x)$ denote the linear function determined by $(a, f(a))$ and $(c, f(c))$. If f is concave up, then $f(b) \leq g(b)$.



We find the equation of the straight line connecting $(a, f(a))$ and $(c, f(c))$. Since it passes through $(a, f(a))$ and has slope $\frac{f(c) - f(a)}{c - a}$, its equation is

$$\begin{aligned} g(x) &= y = \frac{f(c) - f(a)}{c - a} (x - a) + f(a) & \frac{f(c) - f(a)}{c - a} &= m_{ac} \\ g(x) &= m_{ac}(x - a) + f(a) \end{aligned}$$

Now the statement $f(b) \leq g(b)$ becomes

$$\begin{aligned} f(b) &\leq g(b) \\ f(b) &\leq m_{ac}(b - a) + f(a) && \text{subtract } f(a) \\ f(b) - f(a) &\leq m_{ac}(b - a) && \text{recall } a < b \\ \frac{f(b) - f(a)}{b - a} &\leq m_{ac} \\ m_{ab} &\leq m_{ac} \end{aligned}$$

The line connecting $(a, f(a))$ and $(c, f(c))$ can also be written as (Since it also passes through $(c, f(c))$)

$$g(x) = y = m_{ac}(x - c) + f(c)$$

The same statement $f(b) \leq g(b)$ becomes

$$\begin{aligned} f(b) &\leq g(b) \\ f(b) &\leq m_{ac}(b - c) + f(c) && \text{subtract } f(c) \\ f(b) - f(c) &\leq m_{ac}(b - c) && \text{recall } b < c \implies b - c < 0 \\ \frac{f(b) - f(c)}{b - c} &\geq m_{ac} \\ m_{bc} &\geq m_{ac} \end{aligned}$$