

Sample Problems

Compute each of the following integrals.

1. $\int x e^x dx$

2. $\int x e^{-4x} dx$

3. $\int \ln x dx$

Practice Problems

1. $\int x e^{2x} dx$

5. $\int x 2^x dx$

9. $\int_1^9 \frac{\ln x}{\sqrt{x}} dx$

2. $\int x e^{-3x} dx$

6. $\int x^2 2^x dx$

10. $\int_1^\infty \frac{\ln x}{x^7} dx$

3. $\int_0^{\ln 2} x e^{-3x} dx$

7. $\int x \ln x dx$

11. $\int \frac{x^3}{(x^2 + 2)^2} dx$

4. $\int_0^\infty x e^{-3x} dx$

8. $\int x^5 \ln x dx$

Sample Problems - Answers

$$1.) \quad xe^x - e^x + C \qquad 2.) \quad -\frac{1}{16}e^{-4x} - \frac{1}{4}xe^{-4x} + C \qquad 3.) \quad x \ln x - x + C$$

Practice Problems - Answers

$$\begin{aligned} 1.) \quad & \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C & 2.) \quad & -\frac{1}{9}e^{-3x} - \frac{1}{3}xe^{-3x} + C & 3.) \quad & \frac{7}{72} - \frac{1}{24}\ln 2 & 4.) \quad & \frac{1}{9} \\ 5.) \quad & \frac{2^x}{\ln 2} \left(x - \frac{1}{\ln 2} \right) + C & 6.) \quad & \frac{2^x}{\ln 2} \left(x^2 - \frac{2x}{\ln 2} + \frac{2}{\ln^2 2} \right) + C & 7.) \quad & \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \\ 8.) \quad & \frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C & 9.) \quad & 6 \ln 9 - 8 & 10.) \quad & \frac{1}{36} & 11.) \quad & \frac{1}{2} \ln(x^2 + 2) + \frac{1}{x^2 + 2} + C \end{aligned}$$

Sample Problems - Solutions

Compute each of the following integrals.

1. $\int x e^x dx$

Solution: We will integrate this by parts, using the formula

$$\int f'g = fg - \int fg'$$

Let $g(x) = x$ and $f'(x) = e^x$. Then we obtain g' and f by differentiation and integration.

$f(x) = e^x$	$g(x) = x$
$f'(x) = e^x$	$g'(x) = 1$

$$\begin{aligned} \int f'g &= fg - \int fg' \quad \text{becomes} \\ \int x e^x dx &= x e^x - \int e^x dx = x e^x - e^x + C \end{aligned}$$

We should check our result by differentiating the answer. Indeed,

$$(x e^x - e^x + C)' = e^x + x e^x - e^x = x e^x$$

2. $\int x e^{-4x} dx$

Solution: Let $g(x) = x$ and $f'(x) = e^{-4x}$. Then we obtain g' and f by differentiation and integration. To compute $f(x)$, we will use substitution. Let $u = -4x$ then $du = -4dx$ and so $dx = \frac{du}{-4}$.

$$f(x) = \int e^{-4x} dx = \int e^u \frac{du}{-4} = -\frac{1}{4} \int e^u du = -\frac{1}{4} e^u + C = -\frac{1}{4} e^{-4x} + C$$

We will choose $C = 0$ and so $f(x) = -\frac{1}{4} e^{-4x}$.

$f(x) = -\frac{1}{4} e^{-4x}$	$g(x) = x$
$f'(x) = e^{-4x}$	$g'(x) = 1$

$$\begin{aligned} \int f'g &= fg - \int fg' \quad \text{becomes} \\ \int x e^{-4x} dx &= -\frac{1}{4} x e^{-4x} - \int -\frac{1}{4} e^{-4x} dx = -\frac{1}{4} x e^{-4x} + \frac{1}{4} \int e^{-4x} dx = -\frac{1}{4} x e^{-4x} + \frac{1}{4} \left(-\frac{1}{4} e^{-4x} \right) + C \\ &= -\frac{1}{4} x e^{-4x} - \frac{1}{16} e^{-4x} + C \end{aligned}$$

We check our result by differentiating the answer.

$$\begin{aligned}
 & \left(-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C \right)' = \\
 &= -\frac{1}{4}(xe^{-4x})' - \frac{1}{16}(e^{-4x})' = -\frac{1}{4}(e^{-4x} + x(-4e^{-4x})) - \frac{1}{16}(-4e^{-4x}) \\
 &= -\frac{1}{4}e^{-4x} + xe^{-4x} + \frac{1}{4}e^{-4x} = xe^{-4x}
 \end{aligned}$$

3. $\int \ln x \, dx$

Solution: Let $g(x) = \ln x$ and $f'(x) = 1$. Then we obtain g' and f by differentiation and

integration.

$f(x) = x$	$g(x) = \ln x$
$f'(x) = 1$	$g'(x) = \frac{1}{x}$

$$\begin{aligned}
 \int f'g &= fg - \int fg' \quad \text{becomes} \\
 \int \ln x \, dx &= x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - \int 1 \, dx = x \ln x - x + C
 \end{aligned}$$

We check our result by differentiating the answer.

$$(x \ln x - x + C)' = \ln x + x \cdot \frac{1}{x} - 1 = \ln x$$