

1. Sketch the curve defined by the parametric equation given.

a) $x = t \quad y = t^2 \quad -3 \leq t \leq 3$

d) $x = 2 \cos t \quad y = 3 \sin t \quad 0 \leq t \leq 2\pi$

b) $x = -t^2 \quad y = 2t \quad t \in \mathbb{R}$

e) $x = \sec t \quad y = \tan t \quad -\frac{\pi}{4} < t < \frac{\pi}{4}$

c) $x = 3 \cos t \quad y = 3 \sin t \quad 0 \leq t \leq \pi$

f) $x = \sqrt{t+1} \quad y = \sqrt{t} \quad t \geq 0$

2. Find the rectangular equation of each of the curves defined by the parametric equation given.

a) $x = t \quad y = t^2 \quad -3 \leq t \leq 3$

d) $x = 2 \cos t \quad y = 3 \sin t \quad 0 \leq t \leq 2\pi$

b) $x = -t^2 \quad y = 2t \quad t \in \mathbb{R}$

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c) $x = 3 \cos t \quad y = 3 \sin t \quad 0 \leq t \leq \pi$

f) $x = \sqrt{t+1} \quad y = \sqrt{t} \quad t \geq 0$

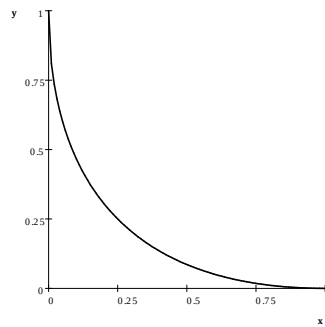
3. Find a parametrization for the line $y = 3x - 1$.

4. Find a parametrization for the line with slope m that passes through the point (a, b) .

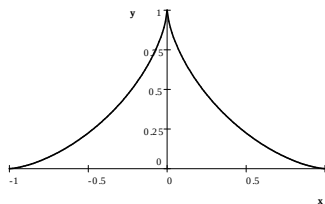
5. Compute the tangent line to the curve $x = t - t^2 \quad y = t - t^3$ at the point $x = (0, 0)$, where $t = 1$.

6. Compute the tangent line to the curve $x = \sec t \quad y = \tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $x = (\sqrt{2}, 1)$, where $t = \frac{\pi}{4}$.

7. Find the area under the curve defined by $x = \cos^4 t \quad y = \sin^4 t \quad t \in \left[0, \frac{\pi}{2}\right]$



8. Find the length of the curve defined by $x = \cos^3 t$ and $y = \sin^3 t \quad t \in [0, \pi]$

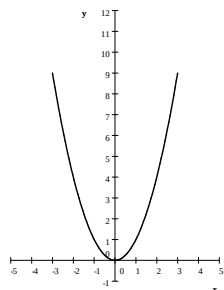


9. Find the length of the curve defined by $x = \cos t$ and $y = \sin t \quad t \in [0, \pi]$

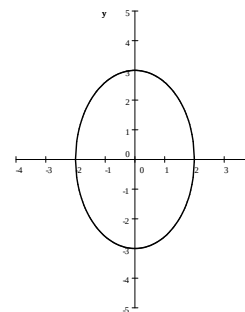
Answers

1. Sketch the curve defined by the parametric equation given.

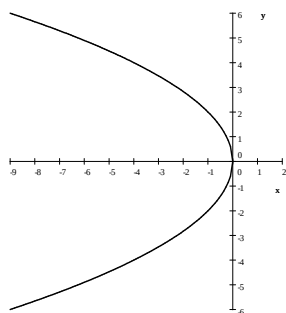
a) $x = t$ $y = t^2$ $-3 \leq t \leq 3$



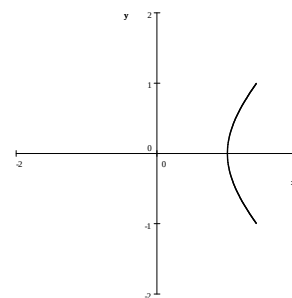
d) $x = 2 \cos t$ $y = 3 \sin t$ $0 \leq t \leq 2\pi$



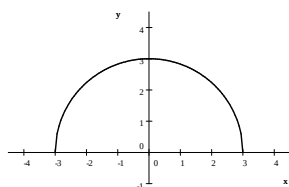
b) $x = -t^2$ $y = 2t$ $t \in \mathbb{R}$



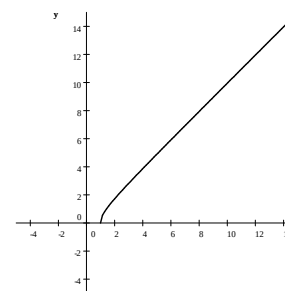
e) $x = \sec t$ $y = \tan t$ $-\frac{\pi}{4} < t < \frac{\pi}{4}$



c) $x = 3 \cos t$ $y = 3 \sin t$ $0 \leq t \leq \pi$



f) $x = \sqrt{t+1}$ $y = \sqrt{t}$ $t \geq 0$



2. Find the regular equation of each of the curves defined by the parametric equation given.

a) $x = t$ $y = t^2$ $-3 \leq t \leq 3$

$y = x^2$ on $[-3, 3]$

c) $x = 3 \cos t$ $y = 3 \sin t$ $0 \leq t \leq \pi$

$y = \sqrt{9 - x^2}$ $x \in [-3, 3]$

b) $x = -t^2$ $y = 2t$ $t \in \mathbb{R}$

d) $x = 2 \cos t$ $y = 3 \sin t$ $0 \leq t \leq 2\pi$

$x = -\frac{y^2}{4}$ $y \in \mathbb{R}$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ $x \in [-2, 2]$

e) $x = \sec t \quad y = \tan t \quad -\frac{\pi}{4} < t < \frac{\pi}{4}.$

$$x^2 - y^2 = 1 \quad 0 < x < \sqrt{2} \quad -1 < y < 1$$

f) $x = \sqrt{t+1} \quad y = \sqrt{t} \quad t \geq 0$

$$x^2 - y^2 = 1 \quad x \geq 1, \quad y \geq 0$$

3. Find a parametrization for the line $y = 3x - 1$.

answers may vary $x = t \quad y = 3t - 1$

4. Find a parametrization for the line with slope m that passes through the point (a, b) .

answers may vary $x = t \quad y = m(t - a) + b$

5. Compute the tangent line to the curve $x = t - t^2 \quad y = t - t^3$ at the point $x = (0, 0)$, where $t = 1$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(t - t^3)}{\frac{d}{dt}(t - t^2)} = \frac{1 - 3t^2}{1 - 2t}$$

At $t = 1$, the slope is $\frac{1 - 3}{1 - 2} = 2$. So the tangent line is $y = 2x$.

6. Compute the tangent line to the curve $x = \sec t \quad y = \tan t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $x = (\sqrt{2}, 1)$, where $t = \frac{\pi}{4}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(\tan t)}{\frac{d}{dt}(\sec t)} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} = \frac{1}{\sin t}$$

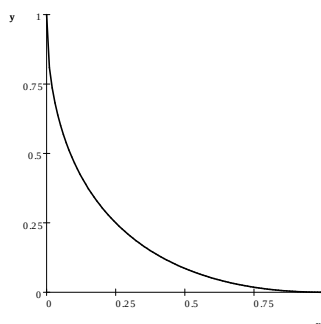
at $t = \frac{\pi}{4}$, that is $\frac{1}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}$. Now that we know the slope is $m = \sqrt{2}$ and the tangent line passes through $(\sqrt{2}, 1)$, the equation is

$$y - 1 = \sqrt{2}(x - \sqrt{2})$$

$$y - 1 = \sqrt{2}x - 2$$

$$y = \sqrt{2}x - 1$$

7. Find the area under the curve defined by $x = \cos^4 t \quad y = \sin^4 t \quad t \in \left[0, \frac{\pi}{2}\right]$



At first, we could say that the area is the integral

$$A = \int_0^1 y dx$$

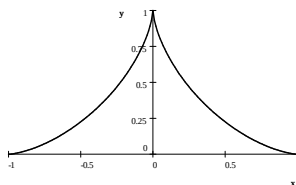
and then we express everything in terms of t . From $x = \cos^4 t$ we obtain that $dx = 4(\cos^3 t)(-\sin t) dt = -4\cos^3 t \sin t dt$ and so the integral becomes

$$\begin{aligned} A &= \int_0^1 y dx = \int_{\pi/2}^0 \sin^4 t (-4\cos^3 t \sin t dt) = \int_{\pi/2}^0 -4\cos^3 t \sin^5 t dt = \int_{\pi/2}^0 -4\cos t (\cos^2 t) \sin^5 t dt \\ &= \int_{\pi/2}^0 -4\cos t (1 - \sin^2 t) \sin^5 t dt = \int_{\pi/2}^0 -4\cos t (\sin^5 t - \sin^7 t) dt \end{aligned}$$

Let $u = \sin t$ then $du = \cos t dt$. When $t = \frac{\pi}{2}$, then $u = 1$ and when $t = 0$, then $u = 0$.

$$\begin{aligned} A &= \int_{\pi/2}^0 -4\cos t (\sin^5 t - \sin^7 t) dt = \int_1^0 -4(u^5 - u^7) du = -4 \left(\frac{u^6}{6} - \frac{u^8}{8} \right) \Big|_1^0 \\ &= -4 \left(0 - \left(\frac{1}{6} - \frac{1}{8} \right) \right) = -4 \left(-\frac{1}{24} \right) = \frac{1}{6} \end{aligned}$$

8. Find the length of the curve defined by $x = \cos^3 t$ and $y = \sin^3 t$ $t \in [0, \pi]$



By symmetry, we will compute the arc length in the first quadrant only, and then multiply the result by 2.

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\frac{dx}{dt} = \frac{d}{dt}(\cos^3 t) = 3 \cos^2 t (-\sin t) = -3 \cos^2 t \sin t \text{ and } \frac{dy}{dt} = \frac{d}{dt}(\sin^3 t) = 3 \sin^2 t (\cos t) = 3 \sin^2 t \cos t$$

$$\begin{aligned} L &= \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{\pi/2} \sqrt{(-3 \cos^2 t \sin t)^2 + (3 \sin^2 t \cos t)^2} dt \\ &= \int_0^{\pi/2} \sqrt{9 \cos^4 t \sin^2 t + 9 \sin^4 t \cos^2 t} dt = \int_0^{\pi/2} \sqrt{9 \cos^2 t \sin^2 t (\sin^2 t + \cos^2 t)} dt \\ &= \int_0^{\pi/2} |3 \cos t \sin t| \sqrt{\cos^2 t + \sin^2 t} dt = \int_0^{\pi/2} 3 \cos t \sin t dt = \int_0^{\pi/2} \frac{3}{2} \sin 2t dt = -\frac{3}{4} \cos 2t \Big|_0^{\pi/2} \\ &= -\frac{3}{4} \left(\cos \left(2 \cdot \frac{\pi}{2} \right) - \cos (2 \cdot 0) \right) = -\frac{3}{4} (\cos \pi - \cos 0) = -\frac{3}{4} (-2) = \frac{3}{2} \end{aligned}$$

So the length of the full arc on $[0, \pi]$ is 3.

9. Find the length of the curve defined by $x = \cos t$ and $y = \sin t$ $t \in [0, \pi]$

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t) = -\sin t \text{ and } \frac{dy}{dt} = \frac{d}{dt}(\sin t) = \cos t$$

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{\pi} \sqrt{\cos^2 t + \sin^2 t} dt = \int_0^{\pi} \sqrt{1} dt = \int_0^{\pi} 1 dt = t \Big|_0^{\pi} = \pi$$