

Practice Problems

Differentiate each of the following functions. (Please note that in many exercises, there is a way to avoid using the quotient rule.)

1. $f(x) = e^{x^2-3x+5}$

9. $f(x) = x^3 + 3^x$

16. $f(x) = \ln \left(\frac{3 - \sin x}{3 + \sin x} \right)^4$

2. $f(x) = \frac{x^2 - 5x + 6}{x - 3}$

10. $f(x) = \frac{e^x}{e^x + 1}$

17. $f(x) = \cos(5^{2x-3})$

3. $f(x) = 2^{\sin x}$

11. $f(x) = \frac{x-2}{x+5}$

18. $f(x) = \frac{2^{5x-1}}{2^{2x+1}}$

4. $f(x) = \log_{10}(x^4 + 8x^2 + e)$

12. $f(x) = 2^{\ln x}$

19. $f(x) = \frac{x^8}{\ln x}$

5. $f(x) = \sin(5^x)$

13. $f(x) = \tan x$

20. $f(x) = e^{\sin x + \cos x}$

7. $f(x) = \frac{1}{2}(e^x - e^{-x})$

14. $f(x) = \frac{x}{x-1}$

21. $f(x) = \tan(e^{-x})$

8. $f(x) = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$

15. $f(x) = \ln(\tan x)$

22. $f(x) = \frac{2^x + 1}{2^x - 1}$

Define $f(x) = \frac{1}{2}(e^x + e^{-x})$ and $g(x) = \frac{1}{2}(e^x - e^{-x})$. These functions have very interesting properties.

23. Compute $f'(x)$.

25. Compute $f(0)$ and $g(0)$.

24. Compute $g'(x)$.

26. Compute $(f(x))^2 - (g(x))^2$.

As we have already seen, these functions have properties similar to trigonometric functions. In fact, $f(x)$ is really called $\cosh x$ (hyperbolic cosine of x) and $g(x)$ is called $\sinh x$ (hyperbolic sine of x)

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) \quad \text{and} \quad \sinh x = \frac{1}{2}(e^x - e^{-x})$$

There are more properties that make them so similar to trigonometric functions.

27. Prove that $\cosh x$ is an even function and $\sinh x$ is an odd function.

28. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$

29. Prove that $2 \sinh x \cosh x = \sinh 2x$

Practice Problems - Answers

1.) $f'(x) = (2x - 3)e^{x^2 - 3x + 5}$

2.) $f'(x) = 1$

3.) $f'(x) = (\ln 2)(\cos x)2^{\sin x}$

4.) $f'(x) = \frac{4x^3 + 16x}{(x^4 + 8x^2 + e)\ln 10}$

5.) $f'(x) = (\ln 5)(5^x)\cos(5^x)$

6.) $f'(x) = -e^{-x}$

7.) $f'(x) = \frac{1}{2}(e^x + e^{-x})$

8.) $f'(x) = xe^{2x}$

9.) $f'(x) = 3x^2 + (\ln 3)3^x$

10.) $f'(x) = \frac{e^x}{(e^x + 1)^2}$

11.) $f'(x) = \frac{7}{(x + 5)^2}$

12.) $f'(x) = \left(\frac{\ln 2}{x}\right)2^{\ln x}$

13.) $f'(x) = \tan^2 x + 1 = \sec^2 x$

14.) $f'(x) = -\frac{1}{(x - 1)^2}$

15.) $f'(x) = \frac{\sec^2 x}{\tan x} = \frac{\tan^2 x + 1}{\tan x} = \tan x + \frac{1}{\tan x} = \tan x + \cot x$

16.) Note: it is easier if we re-write f as $4\ln(3 - \sin x) - 4\ln(3 + \sin x)$

$$f'(x) = -4\cos x \left(\frac{1}{\sin x + 3} + \frac{1}{-\sin x + 3} \right)$$

17.) $f'(x) = -2(\ln 5)(5^{2x-3})\sin(5^{2x-3})$

18.) $f'(x) = 3\ln 2 \cdot 2^{3x-2}$

19.) $f'(x) = \frac{x^7(8\ln x - 1)}{\ln^2 x}$

20.) $f'(x) = e^{\cos x + \sin x}(\cos x - \sin x)$

21.) $f'(x) = -e^{-x}(\tan^2(e^{-x}) + 1)$

22.) $f'(x) = \frac{-2(\ln 2)2^x}{(2^x - 1)^2}$

23.) $f'(x) = \frac{1}{2}(e^x - e^{-x}) = g(x)$

24.) $g'(x) = \frac{1}{2}(e^x + e^{-x}) = f(x)$

25.) $f(0) = 1$ and $g(0) = 0$

26.) 1

27.)

$$\cosh(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)}) = \cosh x \text{ and } \sinh(-x) = \frac{1}{2}(e^{-x} - e^{-(-x)}) = \frac{1}{2}(e^{-x} - e^x) = -\frac{1}{2}(e^x - e^{-x}) = -\sinh x$$

28.)

$$\begin{aligned} \cosh^2 x + \sinh^2 x &= \left[\frac{1}{2}(e^x + e^{-x}) \right]^2 + \left[\frac{1}{2}(e^x - e^{-x}) \right]^2 \\ &= \frac{1}{4}((e^x)^2 + (e^{-x})^2 + 2e^x(e^{-x})) + \frac{1}{4}((e^x)^2 + (e^{-x})^2 - 2e^x(e^{-x})) \\ &= \frac{1}{4}[e^{2x} + e^{-2x} + 2 + e^{2x} + e^{-2x} - 2] = \frac{1}{4}(2e^{2x} + 2e^{-2x}) = \frac{1}{2}(e^{2x} + e^{-2x}) = \cosh 2x \end{aligned}$$

29.)

$$\begin{aligned} 2\sinh x \cosh x &= 2 \left[\frac{1}{2}(e^x - e^{-x}) \right] \left[\frac{1}{2}(e^x + e^{-x}) \right] = 2 \cdot \frac{1}{4}[(e^x - e^{-x})(e^x + e^{-x})] = \frac{1}{2}[(e^x)^2 - (e^{-x})^2] \\ &= \frac{1}{2}[e^{2x} - e^{-2x}] = \sinh 2x \end{aligned}$$

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