

Differentiate each of the following functions.

1. $f(x) = \frac{\sin x}{x}$

4. $f(x) = \frac{x^3 - 1}{x^2}$

8. $f(\theta) = \sec \theta$

2. $f(x) = \frac{x^4 - x^2 + 1}{\cos x}$

5. $f(x) = \frac{1}{x^2 + 1}$

9. $f(x) = \frac{\sqrt{x}}{x^3}$

3. $f(x) = \frac{x + 1}{x - 1}$

6. $f(x) = \frac{x}{\cos x}$

10. $f(x) = \frac{x - 9}{\sqrt{x} + 3}$

7. $f(\theta) = \tan \theta$

11. Preview of calculus 2. A bit more on tangent and secant.

- a) Prove the identity $\tan^2 x + 1 = \sec^2 x$
- b) Based on the identity above, re-write $f'(x)$ when $f(x) = \tan x$.
- c) Compute $f'(x)$ if $f(x) = \sec x$ and re-write it in terms of tangent and/or secant.
- d) Repeat this with co-tangent and co-secant.

Answers

1. $f'(x) = \frac{x \cos x - \sin x}{x^2}$
2. $f'(x) = \frac{(4x^3 - 2x) \cos x + (x^4 - x^2 + 1) \sin x}{\cos^2 x}$
3. $f'(x) = -\frac{2}{(x-1)^2}$
4. $f'(x) = 1 + \frac{2}{x^3}$ or $\frac{x^3 + 2}{x^3}$
5. $f'(x) = \frac{-2x}{(x^2 + 1)^2}$
6. $f'(x) = \frac{\cos x + x \sin x}{\cos^2 x}$
7. $f'(\theta) = \frac{1}{\cos^2 \theta}$
8. $f'(\theta) = \frac{\sin \theta}{\cos^2 \theta}$ Note that this result can also be written as $\frac{\sin \theta}{\cos^2 \theta} = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \tan \theta \sec \theta$
9. $f'(x) = -\frac{5\sqrt{x}}{2x^4}$
10. $f'(x) = \frac{1}{2\sqrt{x}}$ Note that $\frac{x-9}{\sqrt{x}+3} = \sqrt{x} - 3$
11. a) $\tan^2 x + 1 = \sec^2 x$

$$\text{LHS} = \tan^2 x + 1 = \frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$
- b) $f'(x) = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$
- c) $f'(x) = \sec x \tan x$
- d) $(\cot x)' = -\csc^2 x = -1 - \cot^2 x$
- e) $(\csc x)' = -\csc x \cot x$

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