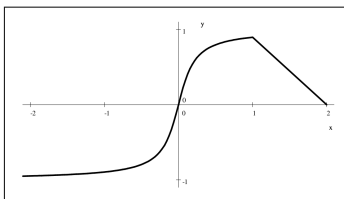


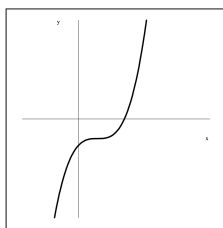
1. Simplify: a) $\frac{x^3 - 9x}{x^2 - 7x + 12}$ b) $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$ c) $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$ d) $\frac{9 - x^{-2}}{3 + x^{-1}}$
2. Rationalize the denominator: a) $\frac{2}{\sqrt{3} + \sqrt{2}}$ b) $\frac{4}{1 - \sqrt{5}}$
3. Assume that all variables represent positive numbers. Write each of the following expressions in the form ca^pb^q where c, p, q are numbers:
 - a) $\frac{(2a^2)^3}{b}$ b) $\sqrt{9ab^3}$ c) $\frac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$ d) $\frac{ab - a}{b^2 - b}$ e) $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$ f) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$
4. Solve for x . Do not use a calculator.
 - a) $5^{x+1} = 25$ b) $\frac{1}{3} = 3^{2x+2}$ c) $\log_2 x = 3$ d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$
5. Simplify a) $\log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$ b) $3^{2\log_3 5}$ c) $2\log_4 9 - \log_2 3$
6. Simplify a) $\log_{10} (10^{1/2})$ b) $\log_{10} \left(\frac{1}{10^x}\right)$ c) $2\log_{10} \sqrt{x} + 3\log_{10} x^{1/3}$
7. Solve the following equations for the indicated variables.
 - a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a b) $S = 2(ab + bc + ac)$, for a
 - c) $A = 2\pi r^2 + 2\pi rh$, for positive r d) $A = P + nPr$, for P
 - e) $2x - 2yd = y + xd$, for d f) $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ for x
8. Find the vertex for each of the following parabolas.
 - a) $y = x^2 + 4x + 3$ b) $3x^2 + 3x + 2y = 0$ c) $9y^2 - 6y - 9 - x = 0$
9. Find all real solutions to each of the following equations.
 - a) $x^6 - 16x^4 = 0$ b) $4x^3 - 8x^2 - 25x + 50 = 0$ c) $8x^3 + 27 = 0$ d) $x^4 = 1$
10. Solve the equations: a) $4x^2 + 12x + 3 = 0$ b) $2x + 1 = \frac{5}{x+2}$ c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$
11. Find the remainders on division of
 - a) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$ b) $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$
12. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
13. Solve the inequalities a) $x^2 + 2x - 3 \leq 0$ b) $\frac{2x-1}{3x-2} \leq 1$ c) $x^2 + x + 1 > 0$
14. Solve for x : a) $|5x - 2| = 8$ b) $|2x + 1| = |x + 3|$
15. Determine the equations of the following lines:
 - a) the line through $(-1, 3)$ and $(2, -4)$
 - b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$
 - c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

16. a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$
 b) Shade the region in the xy -plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$.
17. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find
 a) the center and radius b) the equation of the tangent line at $(-2, 5)$
18. Find the equations of the following circles:
 a) the circle with center at $(1, 2)$ that passes through the point $(-2, -1)$
 b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the x - and y -axes, respectively.
 c) a circle is tangent to the y -axis at $y = 3$ and has one x -intercept at $x = 1$.
19. a) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \frac{5x - 3}{2x + 1}$
 b) Find the domain of the function $f(x) = \frac{3x + 1}{\sqrt{x^2 + x - 2}}$
20. Simplify $\frac{f(x + h) - f(x)}{h}$ where
 a) $f(x) = 2x + 3$ b) $f(x) = x^2$ c) $f(x) = \frac{1}{x + 1}$
21. The graph of a function $y = f(x)$ is given as follows:

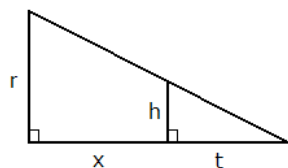


Plot the graph of each of the following functions:

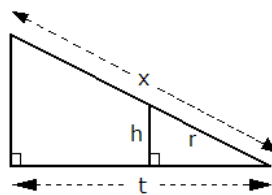
- a) $f(x + 1)$ b) $f(-x)$ c) $|f(x)|$ d) $f(|x|)$
22. Sketch the graph of the functions: a) $g(x) = |3x + 2|$ b) $h(x) = |x(x - 1)|$
23. a) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$
 b) The graph of a quadratic function (a parabola) has x -intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.
24. Find the inverse of the following functions.
 a) $f(x) = 2x + 3$ b) $f(x) = \frac{x + 2}{5x - 1}$ c) $f(x) = x^2 + 2x - 1, \quad x > 0$
25. A function $f(x)$ has the following graph. Sketch the graph of the inverse function $f^{-1}(x)$.



26. Express x in terms of the other variables in the picture.



a)



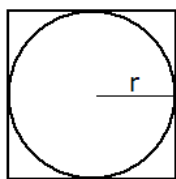
b)

a) $x = t \left(\frac{r-h}{h} \right)$ b) $\frac{rt}{\sqrt{r^2 - h^2}}$

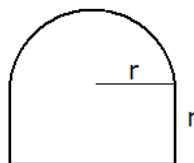
27. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 2)$ is twice its distance from the point $B(2, -1)$. Determine the equation of the curve.

28. Let $f(x) = \frac{|x|}{x}$. Show that $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$. Find the domain and range of $f(x)$.

29. a) Find the ratio of the area inside the square but outside the circle to the area of the square in picture (a) below.



a)



b)

b) Find a formula for the perimeter of a window of the shape on picture (b) above.

c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10 m high and has a radius of 3 m at the top. If the water is 5 m deep (in the middle) what is the surface area of the top of the water?

d) Two cars start moving from the same point. One travels south at $100 \frac{\text{km}}{\text{h}}$ (kilometer per hour), the other west at $50 \frac{\text{km}}{\text{h}}$. How far apart are they two hours later?

e) A kite is 100 m above the ground. If there is 200 m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

30. Without using a calculator, evaluate each of the following:

a) $\cos 210^\circ$ b) $\sin \frac{5\pi}{3}$ c) $\tan^{-1}(-1)$ d) $\sin^{-1}(-1)$

e) $\cos \frac{9\pi}{4}$ f) $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$ g) $\tan \left(\frac{7\pi}{6} \right)$ h) $\cos^{-1}(-1)$

31. Solve for x : a) $3 \sin^2 x = \cos^2 x$; $0 \leq x \leq 2\pi$ b) $\cos^2 x - \sin^2 x = \sin x$; $-\pi \leq x \leq \pi$
 c) $\tan x + \sec x = 2 \cos x$; $-\infty < x < \infty$

32. Assume the following. For all real numbers x and y ,

$$\sin^2 x + \cos^2 x = 1$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \quad \text{and}$$

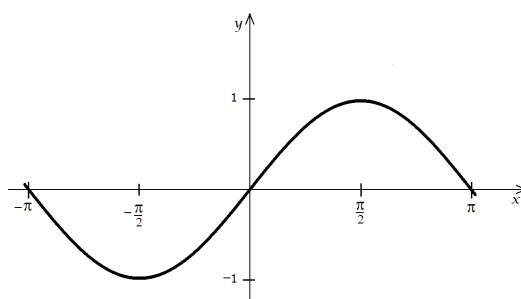
$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

Use the statements above to prove each of the following identities.

a) $\sin 2x = 2 \sin x \cos x$ b) $\cos 2x = \cos^2 x - \sin^2 x$ c) $\cos 2x = 2 \cos^2 x - 1$

d) $\cos 2x = 1 - 2 \sin^2 x$ e) $\left| \cos \frac{x}{2} \right| = \sqrt{\frac{1 + \cos x}{2}}$ f) $\left| \sin \frac{x}{2} \right| = \sqrt{\frac{1 - \cos x}{2}}$

33. Given the graph of $y = \sin x$, sketch the graphs of



a) $y = \sin\left(x - \frac{\pi}{4}\right)$

b) $y = \sin\left(\frac{x}{2}\right)$

c) $y = 2 \sin x$

d) $y = \cos x$

e) $y = \frac{1}{\sin x}$