

1. Differentiate each of the following.

- a) $f(x) = 3x^4 - x^3 + 4x^2 - x + 7$ f) $f(x) = \frac{-x+5}{\sqrt{x^2+1}}$ j) $f(x) = \tan 3x$
 b) $f(x) = \sqrt[5]{2x^6 + 5x^2 + 1}$ g) $f(x) = \frac{x^2+1}{\sin 5x}$ k) $f(x) = \tan(3x^2)$
 c) $f(x) = \cos x + x \sin x$ h) $f(x) = \cot\left(\frac{\pi}{2}x\right)$ l) $f(x) = \sec x + \tan x$
 d) $f(x) = \cos\left(2x - \frac{\pi}{2}\right)$
 e) $f(x) = (3x-1)^{100}$ i) $f(x) = \sqrt{1+\sqrt{x}}$

2. Find the exact value of each of the following.

- a) $\sin\left(\frac{-7\pi}{3}\right)$ b) $\sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$ c) $\tan^{-1}\left(\tan\left(\frac{7\pi}{3}\right)\right)$

3. What is the exact value of $\sin x$ if $\tan x = -2$?

4. Assume that for all real numbers x and y ,

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 \\ \sin(x+y) &= \sin x \cos y + \cos x \sin y \quad \text{and} \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y\end{aligned}$$

Prove each of the following.

- a) $\sin(x-y) = \sin x \cos y - \cos x \sin y$ e) $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
 b) $\cos 2x = 2 \cos^2 x - 1$ f) $\sec^2 x = 1 + \tan^2 x$
 c) $\cos 2x = 1 - 2 \sin^2 x$ g) $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$
 d) $\sin x = \pm \sqrt{\frac{1 - \cos 2x}{2}}$ h) $\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$

5. Simplify each of the following

- a) $\sin(\sin^{-1} x)$ b) $\cos(\sin^{-1} x)$ c) $\sin(\tan^{-1} x)$ d) $\tan(\cos^{-1} x)$

6. Prove that the following expressions are all equivalent.

$$A = \sqrt{\frac{1+\sin x}{1-\sin x}} \quad B = \frac{1+\sin x}{\cos x} \quad C = \sec x + \tan x \quad D = \frac{\cos x}{1-\sin x} \quad E = \frac{1}{\sec x - \tan x}$$

(Hint: prove that $A = B$ and that $B = C$ and that $B = D$ and then $D = E$)

7. Prove each of the following.

- a) $\log_{24} 90 = \frac{\ln 2 + 2 \ln 3 + \ln 5}{3 \ln 2 + \ln 3}$ c) $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 = 3$
 b) $2 \log_{10}(2x) + \log_{10}(25x) - 3 \log_{10}(0.1x) = 5$ d) $\log_3 |\tan x| = -\log_3 |\cot x|$

8. Compute each of the following limits.

a) $\lim_{x \rightarrow \infty} \frac{\sin 5x}{x}$

d) $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$

g) $\lim_{x \rightarrow 0} \frac{\cos x + 1}{x - \pi}$

k) $\lim_{x \rightarrow -1^-} \frac{x^2 + x - 2}{x^2 - 1}$

b) $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

e) $\lim_{x \rightarrow 2} \frac{1}{x^2 - 4}$

h) $\lim_{x \rightarrow 0^+} \log_3 x$

i) $\lim_{x \rightarrow \infty} \tan^{-1} x$

l) $\lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{x^2 - 1}$

c) $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$

f) $\lim_{x \rightarrow 2^-} \frac{1}{x^2 - 4}$

j) $\lim_{x \rightarrow 1^-} \frac{x^2 + x - 2}{x^2 - 1}$

9. Use implicit differentiation to differentiate each of the following.

a) $(x^2 - y^2)^4 = 2xy^2$

b) $(x + y)^3 = \sin x - \sin y$

10. Prove that $\frac{d(x^2 - x)}{dx} = 2x - 1$, using the definition of the derivative as the limit of the differential quotient.

11. Prove that if $f(x) = \sin^{-1} x$ then $f'(x) = \frac{1}{\sqrt{1-x^2}}$

12. Evaluate each of the following integrals.

a) $\int \frac{1}{x^2 + 1} dx$

d) $\int \frac{1}{-x + 2} dx$

g) $\int \frac{1}{x - 5} dx$

b) $\int \frac{x^2}{x^2 + 1} dx$

e) $\int (3x - 1)^{10} dx$

h) $\int 3 - \frac{2}{x - 5} dx$

c) $\int \sin 5x dx$

f) $\int \frac{1}{\sqrt{1 - 9x^2}} dx$

i) $\int \frac{3x - 17}{x - 5} dx$

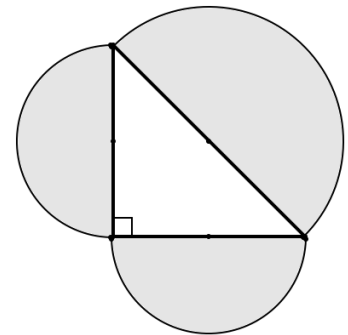
13. Express each of the following in terms of the variable given.

a) Let a be the side of a regular triangle. Express its area in terms of a .

b) Let h be the height of a regular triangle. Express its area in terms of h .

c) Let A be the area of a square. Express its perimeter in terms of A .

d) Let c be the hypotenuse of an isosceles right triangle. We write a semicircle on each of its sides as shown



on the picture below. Express the area of the shaded region in terms of c .

Answers

$$1. \text{ a) } f'(x) = 12x^3 - 3x^2 + 8x - 1 \quad \text{b) } f'(x) = \frac{12x^5 + 10x}{5(2x^6 + 5x^2 + 1)^{4/5}} = \frac{1}{5} \frac{12x^5 + 10x}{2x^6 + 5x^2 + 1} \sqrt[5]{2x^6 + 5x^2 + 1}$$

$$\text{c) } f'(x) = x \cos x \quad \text{d) } f'(x) = -2 \sin \left(2x - \frac{\pi}{2} \right) \quad \text{e) } f'(x) = 300(3x - 1)^{99}$$

$$\text{f) } f'(x) = \frac{-5x - 1}{(x^2 + 1)^{3/2}} \quad \text{g) } f'(x) = \frac{2x \sin 5x - 5(x^2 + 1) \cos 5x}{\sin^2 5x}$$

$$\text{h) } f'(x) = -\frac{\pi}{2} \csc^2 \left(\frac{\pi}{2} x \right) = -\frac{\pi}{2} \left(\cot^2 \frac{\pi}{2} x + 1 \right) \quad \text{i) } f'(x) = \frac{1}{4\sqrt{x}\sqrt{\sqrt{x}+1}}$$

$$\text{j) } f'(x) = 3 \sec^2 3x = 3 \tan^2 3x + 3 \quad \text{k) } f'(x) = 6x (\tan^2 (3x^2) + 1) = 6x \sec^2 (3x^2)$$

$$\text{l) } f'(x) = \sec x \tan x + \sec^2 x = \tan^2 x + 1 + \frac{\sin x}{\cos^2 x}$$

$$2. \text{ a) } -\frac{\sqrt{3}}{2} \quad \text{b) } \frac{\sqrt{3}}{2} \quad \text{c) } \frac{\pi}{3}$$

$$3. \pm \frac{2\sqrt{5}}{5}$$

4. see solutions

$$5. \text{ a) } x \quad \text{b) } \sqrt{1-x^2} \quad \text{c) } \frac{x}{\sqrt{x^2+1}} \quad \text{d) } \frac{\sqrt{1-x^2}}{x}$$

6. see solutions

7. see solutions

$$8. \text{ a) } 0 \quad \text{b) } 5 \quad \text{c) } \frac{1}{6} \quad \text{d) } -\frac{1}{4} \quad \text{e) } \text{undefined} \quad \text{f) } -\infty \quad \text{g) } -\frac{2}{\pi} \quad \text{h) } -\infty \quad \text{i) } \frac{\pi}{2} \quad \text{j) } \frac{3}{2} \\ \text{k) } -\infty \quad \text{l) } 1$$

$$9. \text{ a) } y' = \frac{-y^2 + 4x(x^2 - y^2)^3}{2xy + 4y(x^2 - y^2)^3} \quad \text{b) } y' = \frac{\cos x - 3(x+y)^2}{\cos y + 3(x+y)^2}$$

10. see solutions

11. see solutions

$$12. \text{ a) } \tan^{-1} x + C \quad \text{b) } x - \tan^{-1} x + C \quad \text{c) } -\frac{1}{5} \cos 5x + C \quad \text{d) } -\ln |-x + 2| + C \\ \text{e) } \frac{(3x-1)^{11}}{33} + C \quad \text{f) } \frac{1}{3} \sin^{-1}(3x) + C \quad \text{g) } \ln |x-5| + C \quad \text{h) } 3x - 2 \ln |x-5| + C \\ \text{i) } 3x - 2 \ln |x-5| + C$$

Solution for b):

$$\int \frac{x^2}{x^2+1} dx = \int \frac{x^2+1-1}{x^2+1} dx = \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} dx = \int 1 - \frac{1}{x^2+1} dx = \int 1 dx - \int \frac{1}{x^2+1} dx = x - \tan^{-1} x + C$$

$$13. \text{ a) } \frac{\sqrt{3}}{4} a^2 \quad \text{b) } \frac{\sqrt{3}}{3} h^2 \quad \text{c) } 4\sqrt{A} \quad \text{d) } \frac{1}{4} \pi c^2$$

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