There is a significant difference between **speed** and **velocity**. Speed refers to the 'fastness' of an object's motion without any concern about its direction. For example, if car A is moving to East and car B is moving South but both cars travel exactly 45 miles in one hour, then cars A and B have the same speed, namely 45 $\frac{\text{mi}}{\text{h}}$ (miles per hour). These two cars however will have different velocities as the concept of velocity also includes the direction of movement. Two objects have the same velocity if they move with the same speed, in the same direction, along parallel lines or on the same line.

Speed and velocity usually denoted by v, distance traveled by s, and of course time by t.

To find the average velocity of an object, we need to divide the **displacement** by the time. (The displacement is the directed distance between the starting point and the ending point). To find the average speed of an object, we need to divide the **distance traveled** by the time.

For instance, if an object traveled 10 meters to the right and then 10 meters to the left during 50 seconds, then the displacement is 0 meters and the distance traveled is 20 meters.

$$v_{\text{av}} = \frac{0\,\text{m}}{50\,\text{s}} = 0\frac{\text{m}}{\text{s}} \quad \text{and} \quad sp_{\text{av}} = \frac{20\,\text{m}}{50\,\text{s}} = 0.4\frac{\text{m}}{\text{s}}$$

Sample Problems

1. A car traveled North for 8 hours. After 8 hours, it has traveled a distance of 400 miles. What is the average velocity and average speed of the car?

2. Jesse is driving across the country. On Wednesday, he drove for 8 hours and traveled 384miles. On Thursday, he drove for 3 hours and traveled 123 miles. What was his average speed for the two days? Give both the exact value and an approximation (accurate up to three or more decimal places) of the answer.

3. Erica is driving across the country. On Tuesday, she drove for 3 hours with an average velocity of 59 miles per hour. On Wednesday, she drove in the same direction, for 11 hours with an average velocity of 51 miles per hour. What was her average speed for the two days? Give both the exact value and an approximation (accurate up to three or more decimal places) of the answer.

4. We drove for two days. On the first day, we drove for 4 hours and had an average speed of 50 miles per hour. On the second day, we drove for 6 hours. Our average speed for the two days was 53 miles per hour. What was our average speed on the second day?

5. First we traveled towards North. We covered 200 miles in four hours. Then we traveled toward West for five hours and covered 150 miles.
   a) What was our average speed for the first part of the trip?
   b) What was our average speed for the second part of the trip?
   c) What was our average speed for the entire trip?
   d) What was our average velocity for the first part of the trip?
   e) What was average velocity for the second part of the trip?
   f) What was our average velocity for the entire trip?

6. A bus travels between cities A and B. The distance between these cities is 60 miles. It takes the bus 2 hours to get from A to B. On its way back, the traveling time was only 1.5 hours. Find the average speed of the bus for

   a) the trip from A to B

   b) the trip from B to A

   c) for the roundtrip.

7. A bus travels between cities A and B. From A to B, the bus has an average speed of v_1. On its way back, the average speed is v_2. Express the average speed of the bus in terms of v_1 and v_2.

8*. (Enrichment) A long train is moving slowly, with a constant speed. We walk next to the train, with a constant speed, higher than that of the train. When we walk from the end of the train to the font of it, it takes 200 steps. Then we turn around and walk from the front of the train to the end, and count 120 steps. How many steps long is the train?

Sample Problems- Solutions

1. A car traveled North for 8 hours. After 8 hours, it has traveled a distance of 400 miles. What is the average velocity and average speed of the car?

   Solution: Both average velocity and average speed is

$$v_{\text{av}} = \frac{400\,\text{mi}}{8\,\text{h}} = 50\frac{\text{mi}}{\text{h}}$$

2. Jesse is driving across the country. On Wednesday, he drove for 8 hours and traveled 384miles. On Thursday, he drove for 3 hours and traveled 123 miles. What was his average speed for the two days? Give both the exact value and an approximation (accurate up to three or more decimal places) of the answer.

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1+s_2}{t_1+t_2} = \frac{384\,\text{mi}+123\,\text{mi}}{8\,\text{h}+3\,\text{h}} = \frac{507\,\text{mi}}{11\,\text{h}} = \frac{507}{11}\frac{\text{mi}}{\text{h}}$$

   Exact value: $\frac{507}{11}\frac{\text{mi}}{\text{h}}$ Approximate value: $46.09091\frac{\text{mi}}{\text{h}}$

3. Erica is driving across the country. On Tuesday, she drove for 3 hours with an average velocity of 59 miles per hour. On Wednesday, she drove in the same direction, for 11 hours with an average velocity of 51 miles per hour. What was her average speed for the two days? Give both the exact value and an approximation (accurate up to three or more decimal places) of the answer.

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1+s_2}{t_1+t_2}$$

   But we do not know s_1 and t_1. However, $v = \frac{s}{t}$ implies that $vt = s$. So,

$$s_1 = v_1t_1 = 59\frac{\text{mi}}{\text{h}}\cdot 3\,\text{h} = 177\,\text{mi} \quad \text{and} \quad s_2 = v_2t_2 = 51\frac{\text{mi}}{\text{h}}\cdot 11\,\text{h} = 561\,\text{mi}$$

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1+s_2}{t_1+t_2} = \frac{177\,\text{mi}+561\,\text{mi}}{3\,\text{h}+11\,\text{h}} = \frac{738\,\text{mi}}{14\,\text{h}} = \frac{369}{7}\frac{\text{mi}}{\text{h}}$$

   Exact value: $\frac{369}{7}\frac{\text{mi}}{\text{h}}$ Approximate value: $52.7143\frac{\text{mi}}{\text{h}}$

4. We drove for two days. On the first day, we drove for 4 hours and had an average speed of 50 miles per hour. On the second day, we drove for 6 hours. Our average speed for the two days was 53 miles per hour. What was our average speed on the second day?

   Solution: Recall that $v = \frac{s}{t}$ and $s = vt$

$$
\begin{aligned}
v_{\text{av}} &= \frac{\text{distance traveled}}{\text{time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} \\
v_{\text{av}} &= \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} \\
53\frac{\text{mi}}{\text{h}} &= \frac{50\frac{\text{mi}}{\text{h}} \cdot 4\,\text{h} + v_2 \cdot 6\,\text{h}}{4\,\text{h} + 6\,\text{h}} \\
53 &= \frac{200 + 6v_2}{10} && \text{multiply by 9} \\
530 &= 200 + 6v_2 && \text{subtract 150} \\
330 &= 6v_2 && \text{divide by 6} \\
55 &= v_2
\end{aligned}
$$

   So the average speed for the second day must have been $55\frac{\text{mi}}{\text{h}}$.

5. First we traveled towards North. We covered 200 miles in four hours. Then we traveled toward West for five hours and covered 150 miles.

   a) What was our average speed for the first part of the trip?

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s}{t} = \frac{200\,\text{mi}}{4\,\text{h}} = 50\frac{\text{mi}}{\text{h}}$$

   b) What was our average speed for the second part of the trip?

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s}{t} = \frac{150\,\text{mi}}{5\,\text{h}} = 30\frac{\text{mi}}{\text{h}}$$

   c) What was our average speed for the entire trip?

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{200\,\text{mi} + 150\,\text{mi}}{4\,\text{h} + 5\,\text{h}} = \frac{350\,\text{mi}}{9\,\text{h}} = 38.\overline{8}\frac{\text{mi}}{\text{h}}$$

   d) What was our average velocity for the first part of the trip?

   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s}{t} = \frac{200\,\text{mi}}{4\,\text{h}} = 50\frac{\text{mi}}{\text{h}}$$

   So the average velocity was $50\frac{\text{mi}}{\text{h}}$ to North.

   e) What was average velocity for the second part of the trip?

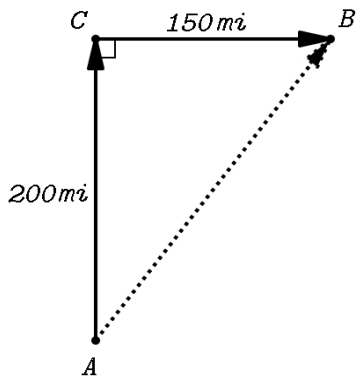
   Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s}{t} = \frac{150\,\text{mi}}{5\,\text{h}} = 30\frac{\text{mi}}{\text{h}}$$

   So the average velocity was $30\frac{\text{mi}}{\text{h}}$ to West.

f) What was our average velocity for the entire trip?

Solution: The displacement involves the distance between the starting and ending point of the trip.



The distance between A and B can be found using the Pythagorean Theorem.

$$\begin{aligned} (AB)^2 &= (200\,\text{mi})^2 + (150\,\text{mi})^2 \\ (AB)^2 &= 40\,000\,\text{mi}^2 + 22\,500\,\text{mi}^2 \\ (AB)^2 &= 62\,500\,\text{mi}^2 \\ AB &= \pm 250\,\text{mi} \quad \Longrightarrow \quad AB = 250\,\text{mi} \end{aligned}$$

The angle at A can also be found, using trigonometry

$$\tan\alpha = \frac{150}{200} = \frac{3}{4} \qquad \alpha = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.869\,9^\circ$$

So the average velocity was

$$v = \frac{s}{t} = \frac{250\,\text{mi}}{9\,\text{h}} \approx 27.\overline{7}\frac{\text{mi}}{\text{h}}$$

So our average velocity was $27.\overline{7}\dfrac{\text{mi}}{\text{h}}$, 36.8699° West from due North.

6. A bus travels between cities A and B. The distance between these cities is 60 miles. It takes the bus 2 hours to get from A to B. On its way back, the traveling time was only 1.5 hours. Find the average speed of the bus for

a) the trip from A to B

Solution:

$$v = \frac{s}{t} = \frac{60\,\text{mi}}{2\,\text{h}} = 30\frac{\text{mi}}{\text{h}}$$

b) the trip from B to A

Solution:

$$v = \frac{s}{t} = \frac{60\,\text{mi}}{1.5\,\text{h}} = 40\frac{\text{mi}}{\text{h}}$$

c) for the roundtrip.

Solution:

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{60\,\text{mi} + 60\,\text{mi}}{2\,\text{h} + 1.5\,\text{h}} = \frac{120}{3.5}\frac{\text{mi}}{\text{h}} \approx 34.28571\frac{\text{mi}}{\text{h}}$$

7. A bus travels between cities A and B. From A to B, the bus has an average speed of v_1. On its way back, the average speed is v_2. Express the average speed of the bus in terms of v_1 and v_2.

   Solution: Recall that $v = \dfrac{s}{t}$ and so $t = \dfrac{s}{v}$

$$v_{\text{av}} = \frac{\text{distance traveled}}{\text{time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2s}{\frac{sv_2}{v_1v_2} + \frac{sv_1}{v_1v_2}} = \frac{2s}{\frac{sv_2 + sv_1}{v_1v_2}}$$

$$= 2s \cdot \frac{v_1v_2}{s\,(v_2 + v_1)} = \frac{2\cancel{s}v_1v_2}{\cancel{s}(v_2 + v_1)} = \frac{2v_1v_2}{v_2 + v_1}$$

