Differentiate each of the following functions.

1. $f(x) = \dfrac{\sin x}{x}$
2. $f(x) = \dfrac{x^4 - x^2 + 1}{\cos x}$
3. $f(x) = \dfrac{x+1}{x-1}$
4. $f(x) = \dfrac{x^3 - 1}{x^2}$
5. $f(x) = \dfrac{1}{x^2+1}$
6. $f(x) = \dfrac{\ln x}{x}$
7. $f(\theta) = \tan\theta$
8. $f(\theta) = \sec\theta$
9. $f(x) = \dfrac{\cos x}{\log_3 x}$
10. $f(x) = \dfrac{\sqrt{x}}{x^3}$
11. $f(x) = \log_2 x + \log_x 2$
12. $f(x) = \dfrac{1+\ln x}{x^2 - \ln x}$
13. Preview of calculus 2. A bit more on tangent and secant.
    a) Prove the identity $\tan^2 x + 1 = \sec^2 x$
    b) Based on the identity above, re-write $f'(x)$ when $f(x) = \tan x$.
    c) Based on the previous answer, find $\displaystyle\int \tan^2 x dx$
    d) Compute $f'(x)$ if $f(x) = \sec x$ and re-write it in terms of tangent and/or secant.

Answers

1.) $f'(x) = \dfrac{x\cos x - \sin x}{x^2}$ 2.) $f'(x) = \dfrac{\left(4x^3 - 2x\right)\cos x + \left(x^4 - x^2 + 1\right)\sin x}{\cos^2 x}$ 3.) $f'(x) = -\dfrac{2}{(x-1)^2}$

4.) $f'(x) = 1 + \dfrac{2}{x^3}$ or $\dfrac{x^3+2}{x^3}$ 5.) $f'(x) = \dfrac{-2x}{\left(x^2+1\right)^2}$ 6.) $f'(x) = \dfrac{1-\ln x}{x^2}$ 7.) $f'(\theta) = \dfrac{1}{\cos^2\theta}$

8.) $f'(\theta) = \dfrac{\sin\theta}{\cos^2\theta}$

Note that the result can also be written as $\dfrac{\sin\theta}{\cos^2\theta} = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\cos\theta} = \tan\theta\sec\theta$

9.) $f'(x) = \dfrac{-\sin x\log_3 x - \cos x\dfrac{1}{x\ln 3}}{\left(\log_3 x\right)^2} = \dfrac{-x\ln 3\sin x\log_3 x - \cos x}{\left(\log_3 x\right)^2 x\ln 3} = \dfrac{-x\sin x\ln x - \cos x}{\left(\log_3 x\right)^2 x\ln 3}$

$$= \dfrac{-x\sin x\ln x - \cos x}{\left(\dfrac{\ln x}{\ln 3}\right)^2 x\ln 3} = \dfrac{-\ln 3\left(x\ln x\sin x + \cos x\right)}{x\left(\ln x\right)^2}$$

10.) $f'(x) = -\dfrac{5\sqrt{x}}{2x^4}$ 11.) $f'(x) = \dfrac{1}{x\ln 2} - \dfrac{\ln 2}{x\ln^2 x}$ 12.) $f'(x) = \dfrac{-x^2 - 2x^2\ln x + 1}{x\left(x^2 - \ln x\right)^2}$

13.) a) $\tan^2 x + 1 = \sec^2 x$

$$\text{LHS} = \tan^2 x + 1 = \frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

b) $f'(x) = \dfrac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$ c) $\displaystyle\int \tan^2 x\,dx = \tan x - x + C$ d) $f'(x) = \sec x\tan x$

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