

Differentiation Formula

$$1. \frac{d}{dx} (\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$2. \frac{d}{dx} (\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$3. \frac{d}{dx} (\arctan x) = \frac{1}{1+x^2}$$

$$4. \frac{d}{dx} (\operatorname{arccot} x) =$$

$$5. \frac{d}{dx} (\tan x) = \sec^2 x = 1 + \tan^2 x$$

$$6. \frac{d}{dx} (\cot x) = -\csc^2 x = -1 - \cot^2 x$$

Integration Formula

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\int -\frac{1}{\sqrt{1-x^2}} dx = \arccos x + C$$

$$\int \frac{1}{x^2+1} dx = \arctan x + C$$

$$4) \int \csc x dx = -\ln |\csc x + \cot x| + C$$

$$5) \int \tan x dx = \ln |\sec x| + C$$

$$\int \cot x dx = \ln |\sin x| + C$$

Proofs

$$1) \quad \frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\begin{aligned} \sin(\sin^{-1} x) &= x \\ \frac{d(\sin(\sin^{-1} x))}{dx} &= \frac{d(x)}{dx} \\ \cos(\sin^{-1} x) \cdot \frac{d(\sin^{-1} x)}{dx} &= 1 \\ \frac{d(\sin^{-1} x)}{dx} &= \frac{1}{\cos(\sin^{-1} x)} \end{aligned}$$

We now only need to simplify $\cos(\sin^{-1} x)$. Let $\beta = \sin^{-1} x$. This means that $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$, $\sin \beta = x$, and we need to simplify $\cos \beta$. Since $\sin^2 \beta + \cos^2 \beta = 1$,

$$\cos \beta = \pm \sqrt{1 - \sin^2 \beta} = \pm \sqrt{1 - x^2}$$

Since $-\frac{\pi}{2} < \beta < \frac{\pi}{2}$, $\cos \beta$ is positive and so $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$. Thus $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1 - x^2}}$.

$$2) \quad \frac{d(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\begin{aligned} \cos(\cos^{-1} x) &= x \\ \frac{d(\cos(\cos^{-1} x))}{dx} &= \frac{d(x)}{dx} \\ -\sin(\cos^{-1} x) \cdot \frac{d(\cos^{-1} x)}{dx} &= 1 \\ \frac{d(\cos^{-1} x)}{dx} &= -\frac{1}{\sin(\cos^{-1} x)} \end{aligned}$$

We now only need to simplify $\sin(\cos^{-1} x)$. Let $\beta = \cos^{-1} x$. This means that $0 < \beta < \pi$, $\cos \beta = x$, and we need to simplify $\sin \beta$. Since $\sin^2 \beta + \cos^2 \beta = 1$,

$$\sin \beta = \pm \sqrt{1 - \cos^2 \beta} = \pm \sqrt{1 - x^2}$$

Since $0 < \beta < \pi$, $\sin \beta$ is positive and so $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$. Thus $\frac{d(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1 - x^2}}$.

$$3) \quad \frac{d(\tan^{-1} x)}{dx} = \frac{1}{x^2 + 1}$$

$$\begin{aligned}
\tan(\tan^{-1} x) &= x \\
\frac{d(\tan(\tan^{-1} x))}{dx} &= \frac{d(x)}{dx} \\
(1 + \tan^2(\tan^{-1} x)) \cdot \frac{d(\tan^{-1} x)}{dx} &= 1 \\
(1 + x^2) \cdot \frac{d(\tan^{-1} x)}{dx} &= 1 \\
\frac{d(\tan^{-1} x)}{dx} &= \frac{1}{x^2 + 1}
\end{aligned}$$

$$4) \quad \frac{d(\cot^{-1} x)}{dx} = -\frac{1}{x^2 + 1}$$

$$\begin{aligned}
\cot(\cot^{-1} x) &= x \\
\frac{d(\cot(\cot^{-1} x))}{dx} &= \frac{d(x)}{dx} \\
(-1 - \cot^2(\cot^{-1} x)) \cdot \frac{d(\cot^{-1} x)}{dx} &= 1 \\
(-1 - x^2) \cdot \frac{d(\cot^{-1} x)}{dx} &= 1 \\
\frac{d(\cot^{-1} x)}{dx} &= -\frac{1}{x^2 + 1}
\end{aligned}$$