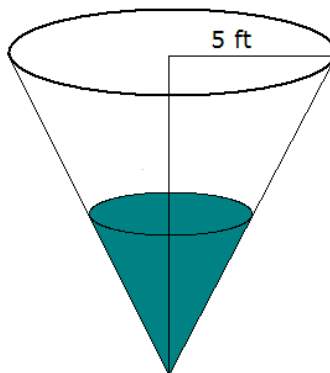


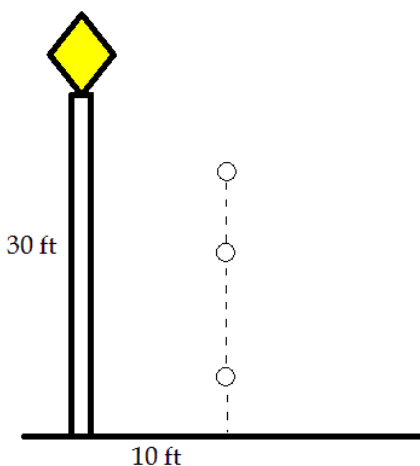
## Sample Problems

1. A city is of a circular shape. The area of the city is growing at a constant rate of  $2\frac{\text{mi}^2}{\text{y}}$  (square miles per year). How fast is the radius growing when it is exactly 15 mi?
2. A sphere is growing in such a manner that its radius increases at  $0.2\frac{\text{m}}{\text{s}}$  (meters per second). How fast is its volume increasing when its radius is 4 m long?
3. A sphere is growing in such a manner that its volume increases at  $0.2\frac{\text{m}^3}{\text{s}}$  (cubic meters per second). How fast is its radius increasing when it is 7 m long?
4. The area of a rectangle is kept fixed at 100 square meters while the lengths of the sides vary. Express the rate of change of the length of the vertical side in terms of the rate of change in the length of the other side when
  - a) the horizontal side is 18 meters long
  - b) the rectangle is a square
5. A ladder 20 ft long leans against a vertical building. If the top of the ladder slides down at a rate of  $\sqrt{3}\frac{\text{ft}}{\text{s}}$ , how fast is the bottom of the ladder sliding away from the building when the top of the ladder is 10 ft above the ground?
6. A tank, shaped like a cone shown on the picture below, is being filled up with water. The top of the tank is a circle with radius 5 ft, its height is 15 ft. Water is added to the tank at the rate of  $V'(t) = 2\pi\frac{\text{ft}^3}{\text{min}}$ . How fast is the water level rising when the water level is 6 ft high? (The volume of a cone with height  $h$  and base radius  $r$  is  $V = \frac{\pi r^2 h}{3}$ .)



7. Two quantities  $p$  and  $q$  depending on  $t$  are subject to the relation  $\frac{1}{p} + \frac{1}{q} = 1$ .
  - a) Express  $p'(t)$  in terms of  $q'(t)$ .
  - b) At a certain moment,  $p(t_0) = \frac{4}{3}$  and  $p'(t_0) = 2$ . Find  $q(t_0)$  and  $q'(t_0)$ .
8. The base radius and height of a cylinder are constantly changing but the volume of the cylinder is kept at a constant  $600\pi \text{ in}^3$ .
  - a) At a time  $t_1$  the base radius is  $r(t_1) = 10 \text{ in}$  and its rate of change is  $r'(t_1) = 0.2\frac{\text{in}}{\text{s}}$ . Compute the rate of change of the height of the cylinder  $h(t)$  at time  $t_1$ .
  - b) At a time  $t_2$  the height is  $h(t_2) = 12 \text{ in}$  and its rate of change is  $h'(t_2) = -0.5\frac{\text{in}}{\text{s}}$ . Compute the rate of change of the radius of the cylinder  $r(t)$  at time  $t_2$ .

9. An object, dropped from a height of  $h$  has a location of  $y(t) = -16t^2 + h$  feet after  $t$  seconds. We dropped a small object from a height of 60 feet.



- Where is the object and what is its velocity after 1.5 seconds?
- Suppose there is a 30 feet tall street light 10 feet away from the point where the object will land. How far is the shadow of the object from the base of the street light at  $t = 1.5$ ?
- How fast is the object's shadow moving at  $t = 1.5$ ?

## Sample Problems - Answers

$$1.) \frac{1}{15\pi} \frac{\text{mi}}{\text{y}} \quad 2.) 12.8\pi \frac{\text{m}^3}{\text{s}} \approx 40.2123859659494 \frac{\text{m}^3}{\text{s}} \quad 3.) \frac{1}{980\pi} \frac{\text{m}}{\text{s}} \approx 3.24806 \times 10^{-4} \frac{\text{m}}{\text{s}}$$

$$4.) \text{ a) } v'(t) = -\frac{25}{81} h'(t) \quad \text{b) } v'(t) = -h'(t) \quad 5.) 1 \frac{\text{ft}}{\text{s}} \quad 6.) \frac{1}{2} \frac{\text{ft}}{\text{min}}$$

$$7.) \text{ a) } p' = -\frac{p^2}{q^2} q' \quad \text{b) } -18 \quad 8.) \text{ a) } -0.24 \frac{\text{in}}{\text{s}} \quad \text{b) } \frac{5}{48} \sqrt{2} \frac{\text{in}}{\text{s}} \approx 0.147314 \frac{\text{in}}{\text{s}}$$

$$9.) \text{ a) } y(1.5) = 24 \text{ ft and } y'(1.5) = -48 \frac{\text{ft}}{\text{s}} \quad \text{b) } 50 \text{ ft} \quad \text{c) } -400 \frac{\text{ft}}{\text{s}}$$