1. a) A city is of a circular shape. The area of the city is growing at a constant rate of $2\frac{\text{mi}^2}{\text{y}}$ (square miles per year). How fast is the radius growing when it is exactly 15 mi?

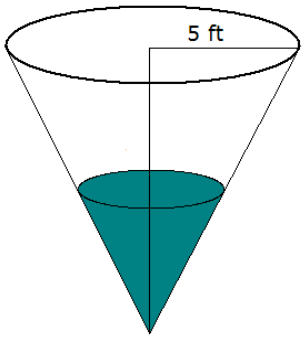
   b) A city is of a circular shape. The radius of the city is growing at a constant rate of $0.3\frac{\text{mi}}{\text{y}}$ (miles per year). How fast is the area growing at the time when the radius of the circle is exactly 8 mi?

2. A sphere is growing in such a manner that its radius increases at $0.2\frac{\text{m}}{\text{s}}$ (meters per second). How fast is its volume increasing when its radius is 4 m long?

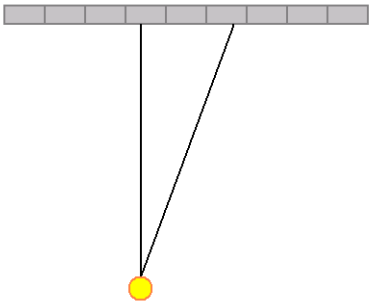
3. A sphere is growing in such a manner that its volume increases at $0.2\frac{\text{m}^3}{\text{s}}$ (cubic meters per second). How fast is its radius increasing when it is 7 m long?

4. A cube is decreasing in size so that its surface is changing at a constant rate of $-0.5\frac{\text{m}^2}{\text{min}}$. How fast is the volume of the cube changing when it is $27\,\text{m}^3$?

5. A tank, shaped like a cone shown on the picture, is being filled up with water. The top of the tank is a circle with radius 5 ft, its height is 15 ft. Water is added to the tank at the rate of $V'(t) = 2\pi\frac{\text{ft}^3}{\text{min}}$. How fast is the water level rising when the water level is 6 ft high? (The volume of a cone with height h and base radius r is $V = \frac{\pi r^2 h}{3}$.)



6. A rotating light is located 20 feet from a wall. The light completes one rotation every 5 seconds. Find the rate at which the light projected onto the wall is moving along the wall when the light's angle is 15 degrees from perpendicular to the wall.



7. The altitude of a triangle is increasing at a rate of 2.2 centimeters/minute while the area of the triangle is increasing at a rate of 1.5 square centimeters/minute. At what rate is the base of the triangle changing when the altitude is 11 centimeters and the area is 87 square centimeters?

8. The area of a rectangle is kept fixed at 100 square meters while the legths of the sides vary. Express the rate of change of the length of the vertical side in terms of the rate of change in the length of the other side when

   a) the horizontal side is 18 meters long b) the rectangle is a square

9. Two quantities p and q depending on t are subject to the relation $\frac{1}{p} + \frac{1}{q} = 1$.

   a) Express $p'(t)$ in terms of $q'(t)$.

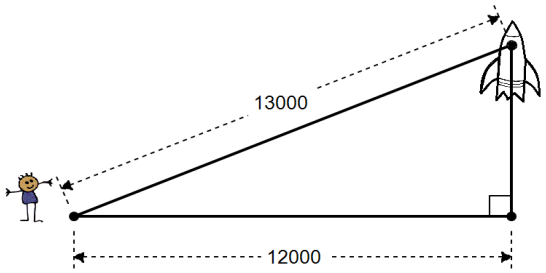
   b) At a certain moment, $p(t_0) = \frac{4}{3}$ and $p'(t_0) = 2$. Find $q(t_0)$ and $q'(t_0)$.

10. The base radius and height of a cylinder are constantly changing but the volume of the cylinder is kept at a constant 600π in^3.

    a) At a time t_1 the base radius is $r(t_1) = 10$ in and its rate of change is $r'(t_1) = 0.2\frac{\text{in}}{\text{s}}$. Compute the rate of change of the height of the cylinder $h(t)$ at time t_1.

    b) At a time t_2 the height is $h(t_2) = 12$ in and its rate of change is $r'(t_2) = -0.5\frac{\text{in}}{\text{s}}$. Compute the rate of change of the radius of the cylinder $r(t)$ at time t_2.

11. At a distance of 12 000 meters from the launch site, a spectator is observing a rocket being launched vertically. What is the speed of the rocket at the instant when the distance of the rocket from the spectator is 13 000 meters and is increasing at the rate of 480 meters per second?



12. A ladder 20 ft long leans against a vertical building. If the top of the ladder slides down at a rate of $\sqrt{3}\frac{\text{ft}}{\text{s}}$, how fast is the bottom of the ladder sliding away from the building when the top of the ladder is 10 ft above the ground?

13. At noon, ship A is 50 miles due west of ship B. Ship A is sailing west at 24 miles per hour and ship B is sailing north at 18 miles per hour. How fast is the distance between the ships changing at 4 PM?

Answers

1.) a) $\frac{1}{15\pi}\frac{\text{mi}}{\text{y}}$ b) $4.8\pi\frac{\text{mi}^2}{\text{y}}$ 2.) $12.8\pi\frac{\text{m}^3}{\text{s}} \approx 40.212\,386\frac{\text{m}^3}{\text{s}}$ 3.) $\frac{1}{980\pi}\frac{\text{m}}{\text{s}} \approx 3.248\,06\times 10^{-4}\frac{\text{m}}{\text{s}}$

4.) $-0.375\frac{\text{m}^3}{\text{min}}$ 5.) $\frac{1}{2}\frac{\text{ft}}{\text{min}}$ 6.) $8\pi\sec^2(15^\circ)\frac{\text{ft}}{\text{s}} \approx 26.93719\frac{\text{ft}}{\text{s}}$ 7.) $-2.89091\frac{\text{cm}}{\text{min}}$

8.) a) $v'(t) = -\frac{25}{81}h'(t)$ b) $v'(t) = -h'(t)$ 9.) a) $p' = -\frac{p^2}{q^2}q'$ b) -18

10.) a) $-0.24\frac{\text{in}}{\text{s}}$ b) $\frac{5}{48}\sqrt{2}\frac{\text{in}}{\text{s}} \approx 0.147\,314\frac{\text{in}}{\text{s}}$ 11.) 1248 meters per second 12.) $1\frac{\text{ft}}{\text{s}}$

13.) 29.4861656 miles per hour

