

1. Consider the function $f(x) = x^2$ on the interval $[0, 6]$.

a) Compute the left Riemann sum for f on this interval with $n = 6$ subintervals.

$$\sum_{j=0}^5 \left(\left(\frac{6}{6} \right) (j) \right)^2 = 55$$

b) Compute the right Riemann sum for f on this interval with $n = 6$ subintervals!

$$\sum_{j=1}^6 \left(\left(\frac{6}{6} \right) (j) \right)^2 = 91$$

c) Compute the left Riemann sum for f on this interval with $n = 12$ subintervals!

$$\sum_{j=0}^{11} \left(\left(\frac{6}{12} \right) (j) \right)^2 \left(\frac{6}{12} \right) = \frac{253}{4} = 63.25$$

d) Compute the right Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=1}^{12} \left(\left(\frac{6}{12} \right) (j) \right)^2 \left(\frac{6}{12} \right) = \frac{325}{4} = 81.25$$

e) Compute the left Riemann sum for f on this interval with $n = 100$ subintervals.

$$\sum_{j=0}^{99} \left(\left(\frac{6}{100} \right) (j) \right)^2 \left(\frac{6}{100} \right) = \frac{177\,309}{2500} = 70.924$$

f) Compute the right Riemann sum for f on this interval with $n = 100$ subintervals.

$$\sum_{j=1}^{100} \left(\left(\frac{6}{100} \right) (j) \right)^2 \left(\frac{6}{100} \right) = \frac{182\,709}{2500} = 73.084$$

g) Compute the left Riemann sum for f on this interval with n subintervals.

$$\sum_{j=0}^{n-1} \left(\left(\frac{6}{n} \right) (j) \right)^2 \left(\frac{6}{n} \right) = \frac{36(2n-1)(n-1)}{n^2} = \frac{72n^2 - 108n + 36}{n^2}$$

h) Compute the limit of the left Riemann sum for f on this interval with n intervals, as n approaches infinity.

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i) Compute the right Riemann sum for f on this interval with n subintervals.

$$\sum_{j=1}^n \left(\left(\frac{6}{n} \right) (j) \right)^2 \left(\frac{6}{n} \right) = \frac{36(n+1)(2n+1)}{n^2} = \frac{72n^2 + 108n + 36}{n^2}$$

j) Compute the limit of the right Riemann sum for f on this interval with n intervals, as n approaches infinity.

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2. Consider the function $f(x) = \sqrt{x}$ on the interval $[0, 4]$.

a) Compute the left Riemann sum for f on this interval with $n = 4$ subintervals.

$$\sum_{j=0}^3 \sqrt{\left(\frac{4}{4} \right) (j)} = \sqrt{2} + \sqrt{3} + 1 = 4.146\,264\,369\,941\,97$$

b) Compute the right Riemann sum for f on this interval with $n = 4$ subintervals.

$$\sum_{j=1}^4 \sqrt{\left(\frac{4}{4}\right)(j)} = \sqrt{2} + \sqrt{3} + 3 = 6.146\,264\,369\,941\,97$$

c) Compute the left Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=0}^9 \left(\frac{4}{10}\right) \sqrt{\frac{4}{10}j} = \left(\frac{4}{10}\right) \sqrt{\frac{4}{10}} \sum_{j=0}^9 \sqrt{j} =$$

$$\frac{2\sqrt{10}}{25} (\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + 6) \approx 4.884\,075$$

d) Compute the right Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=1}^{10} \left(\frac{4}{10}\right) \sqrt{\frac{4}{10}j} = \left(\frac{4}{10}\right) \sqrt{\frac{4}{10}} \sum_{j=1}^{10} \sqrt{j} =$$

$$\frac{2\sqrt{10}}{25} (\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{10} + 6) \approx 5.684\,075$$

3. Consider the function $f(x) = \ln(x+1)$ on the interval $[0, 10]$.

a) Compute the left Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=0}^9 \ln(j+1) = \ln(10!) = \ln 3\,628\,800 = 15.104\,412\,573\,075\,5$$

b) Compute the right Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=1}^{10} \ln(j+1) = \ln(11!) = \ln 39\,916\,800 = 17.502\,307\,845\,873\,9$$

4. Consider the function $f(x) = x^3$ on the interval $[0, 1]$.

a) Compute the left Riemann sum for f on this interval with $n = 4$ subintervals.

$$\sum_{j=0}^3 \left(\left(\frac{1}{4}\right)(j)\right)^3 \left(\frac{1}{4}\right) = \frac{9}{64} = 0.140\,625$$

b) Compute the right Riemann sum for f on this interval with $n = 4$ subintervals!

$$\sum_{j=1}^4 \left(\left(\frac{1}{4}\right)(j)\right)^3 \left(\frac{1}{4}\right) = \frac{25}{64} = 0.390\,625$$

c) Compute the left Riemann sum for f on this interval with $n = 10$ subintervals.

$$\sum_{j=0}^9 \left(\left(\frac{1}{10}\right)(j)\right)^3 \left(\frac{1}{10}\right) = \frac{81}{400} = 0.202\,5$$

d) Compute the right Riemann sum for f on this interval with $n = 10$ subintervals!

$$\sum_{j=1}^{10} \left(\left(\frac{1}{10}\right)(j)\right)^3 \left(\frac{1}{10}\right) = \frac{121}{400} = 0.302\,5$$

e) Compute the left Riemann sum for f on this interval with $n = 100$ subintervals.

$$\sum_{j=0}^{99} \left(\left(\frac{1}{100}\right)(j)\right)^3 \left(\frac{1}{100}\right) = \frac{9801}{40\,000} = 0.245\,025$$

f) Compute the right Riemann sum for f on this interval with $n = 100$ subintervals!

$$\sum_{j=1}^{100} \left(\left(\frac{1}{100} \right)^{(j)} \right)^3 \left(\frac{1}{100} \right) = \frac{10\,201}{40\,000} = 0.255\,025$$

Differentiate each of the following functions.

Answers