So far, our experiments had finitely many outcomes, and we computed probabilities by counting: $P(E) = \dfrac{n(E)}{n(S)}$. But some experiments have infinitely many outcomes. A bus can arrive at *any* moment, and a dart can land at *any* point of a board. We cannot count these outcomes, but we can measure them: instead of counting, we compare lengths or areas.

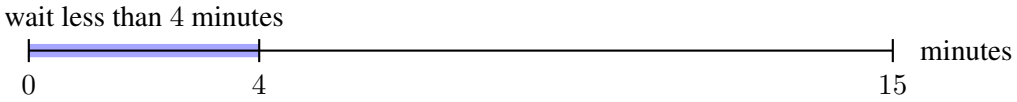
Geometric Probability. Suppose that a point is selected at random from a region S, so that every point of S is equally likely. Then the probability that the point lands in a part E of the region is

$$P(E) = \frac{\text{area of } E}{\text{area of } S}.$$

If the region is a line segment or a time interval, we use lengths instead of areas.

Example 1. A bus comes to a bus stop every 15 minutes. You arrive at the bus stop at a random time, without checking the schedule. What is the probability that you will wait less than 4 minutes?

Solution. Your waiting time can be any number between 0 and 15 minutes, and all of these are equally likely. You wait less than 4 minutes if your waiting time is in the first 4 minutes of the interval.

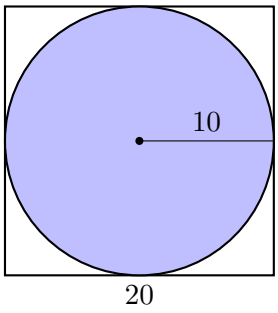


So the probability is

$$P(\text{wait less than } 4 \text{ minutes}) = \frac{4}{15} \approx 0.267.$$

Example 2. A dart hits a square board that is 20 inches wide at a random point. A circle with a radius of 10 inches is painted inside the square, touching all four sides. Find the probability that the dart lands inside the circle.

Solution. The whole board is the sample space. Its area is $20 \cdot 20 = 400$ square inches. The area of the circle is $\pi r^2 = \pi \cdot 10^2 = 100\pi$ square inches.

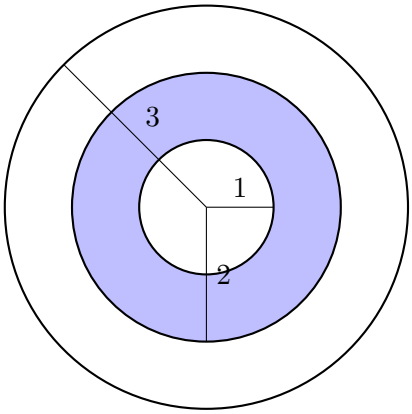


$$P(\text{inside the circle}) = \frac{100\pi}{400} = \frac{\pi}{4} \approx 0.785$$

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Example 3. A target consists of three circles with the same center. Their radii are 1, 2, and 3 feet. An arrow hits the target at a random point. Find the probability that the arrow lands in the middle ring, between the smallest and the middle circle.

Solution. The area of the whole target is $\pi \cdot 3^2 = 9\pi$. The middle ring is the middle circle with the smallest circle removed, so its area is $\pi \cdot 2^2 - \pi \cdot 1^2 = 4\pi - \pi = 3\pi$.



$$P(\text{middle ring}) = \frac{3\pi}{9\pi} = \frac{1}{3}$$

Notice that π cancels, and the answer does not depend on the units.

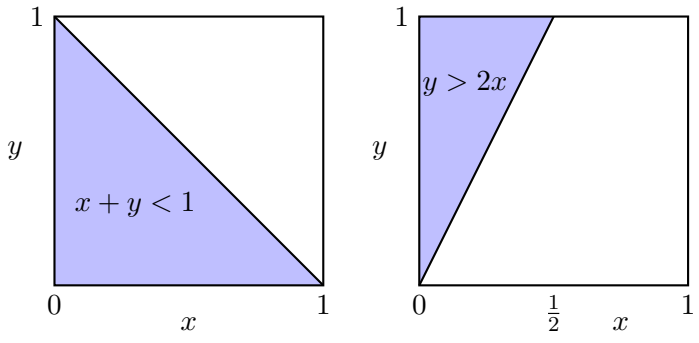
Two Random Numbers

When an experiment produces *two* random numbers, x and y, we can think of the pair (x, y) as a random point in the plane. This turns many problems into area problems.

Example 4. Two numbers, x and y, are selected at random between 0 and 1. Find the probability that

a) $x + y < 1$, b) $y > 2x$.

Solution. The point (x, y) is a random point of the unit square, which has an area of 1. So the probability of an event is simply the area of the region where the event happens.



a) The region below the line $x + y = 1$ is a right triangle with legs 1 and 1. Its area is $\dfrac{1 \cdot 1}{2} = \dfrac{1}{2}$, so $P(x + y < 1) = \dfrac{1}{2}$.

b) The region above the line $y = 2x$ is a right triangle with legs $\dfrac{1}{2}$ and 1. Its area is $\dfrac{1}{2} \cdot \dfrac{1}{2} \cdot 1 = \dfrac{1}{4}$, so $P(y > 2x) = \dfrac{1}{4}$.

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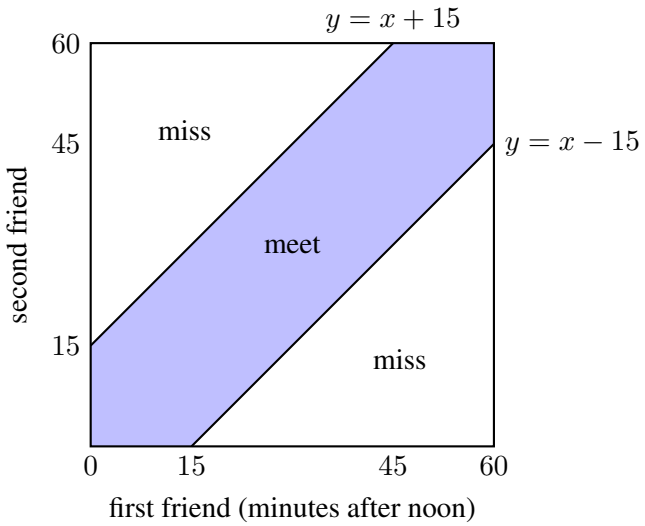
Meeting Problems

Example 5. Two friends agree to meet at a coffee shop sometime between 12 noon and 1 PM. Each of them arrives at a random time during this hour, and each is willing to wait 15 minutes for the other before leaving. What is the probability that they will meet?

Solution. Let x and y be the arrival times of the two friends, in minutes after noon. The point (x, y) is a random point of a 60 by 60 square, whose area is 3 600. The friends meet if their arrival times differ by at most 15 minutes:

$$x - 15 \le y \le x + 15$$

This is the shaded band around the diagonal.



It is easier to compute the area of the two unshaded triangles, where the friends miss each other. Each is a right triangle with legs 45 and 45, so together their area is $2 \cdot \dfrac{45 \cdot 45}{2} = 2\,025$. Therefore

$$P(\text{meet}) = \frac{3\,600 - 2\,025}{3\,600} = \frac{1\,575}{3\,600} = \frac{7}{16} = 0.4375.$$

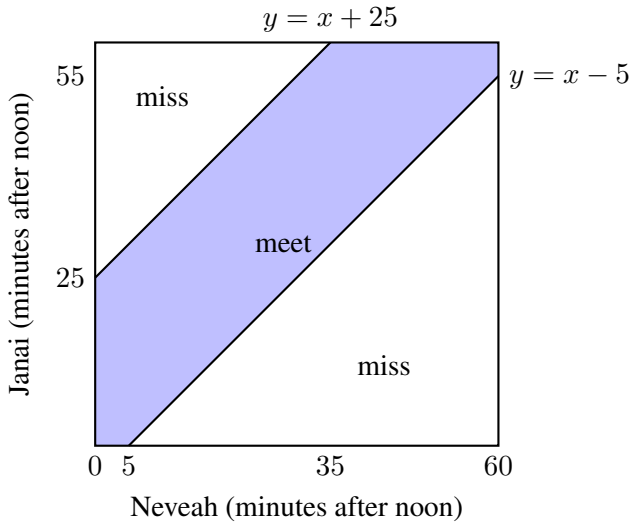
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Example 6. Neveah and Janai agreed to maybe meet at the library sometime between 12 noon and 1 PM. Suppose they can both arrive at any time during this hour with equal likelihood. Neveah is willing to wait 25 minutes before leaving. Janai is willing to wait 5 minutes before leaving. What is the probability that they will meet? Round your answer to three decimal places.

Solution. Let x be Neveah's arrival time and y be Janai's arrival time, in minutes after noon. Again, (x, y) is a random point of a 60 by 60 square. This time the waiting times are different, so we must consider who arrives first.

If Neveah arrives first, she waits 25 minutes. They meet if Janai arrives at most 25 minutes after Neveah: $y \le x + 25$.

If Janai arrives first, she waits only 5 minutes. They meet if Neveah arrives at most 5 minutes after Janai: $x \le y + 5$, or $y \ge x - 5$.



The two unshaded triangles are right triangles. The upper one has legs 35 and 35, and the lower one has legs 55 and 55. Their total area is

$$\frac{35 \cdot 35}{2} + \frac{55 \cdot 55}{2} = 612.5 + 1\,512.5 = 2\,125.$$

So the area where they meet is $3\,600 - 2\,125 = 1\,475$, and

$$P(\text{meet}) = \frac{1\,475}{3\,600} \approx 0.410.$$

Be careful. Always check which line belongs to which person. The person who arrives first is the one who waits, so Neveah's 25 minutes give the line above the diagonal, where $y > x$ (Janai arrives later).

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Practice Problems

1. A train comes to a station every 20 minutes. You arrive at the station at a random time. Find the probability that you will wait

   a) at most 6 minutes, b) more than 15 minutes.

2. A point is selected at random on a line segment that is 12 inches long. Find the probability that the point is closer to the midpoint of the segment than to either endpoint.

3. A point is selected at random inside a square with sides of 10 cm. Find the probability that the point is

   a) inside the circle that touches all four sides of the square,

   b) within 2 cm of the lower left corner of the square.

4. A dartboard is a circle with a radius of 12 inches. The bull's-eye is a circle with a radius of 1.5 inches, in the middle of the board. A dart hits the board at a random point. Find the probability that it hits the bull's-eye.

5. Two numbers, x and y, are selected at random between 0 and 1. Find the probability that

   a) $x + y > 1.5$, b) x and y differ by less than 0.2.

6. Ana and Ben agree to meet at a park sometime between 3 PM and 4 PM. Each arrives at a random time during this hour, and each is willing to wait 10 minutes for the other. What is the probability that they will meet? Round your answer to three decimal places.

7. Malik and Sofia agree to meet at a restaurant sometime between 6 PM and 7 PM. Each arrives at a random time during this hour. Malik is willing to wait 20 minutes, and Sofia is willing to wait 10 minutes. What is the probability that they will meet? Round your answer to three decimal places.

8. Priya and Tom plan to meet at the mall sometime between 2 PM and 3 PM. Each arrives at a random time during this hour. Priya is willing to wait 15 minutes, but Tom does not wait at all: if Priya is not there when he arrives, he leaves right away. What is the probability that they will meet? Round your answer to three decimal places.

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Answers

1. a) $\frac{6}{20} = 0.3$ b) $\frac{5}{20} = 0.25$

2. $\frac{6}{12} = \frac{1}{2}$ (the point must be within 3 inches of the midpoint)

3. a) $\frac{25\pi}{100} = \frac{\pi}{4} \approx 0.785$ b) $\frac{\frac{1}{4} \cdot \pi \cdot 2^2}{100} = \frac{\pi}{100} \approx 0.031$

4. $\frac{\pi \cdot 1.5^2}{\pi \cdot 12^2} = \frac{2.25}{144} = \frac{1}{64} \approx 0.016$

5. a) $\frac{0.5 \cdot 0.5}{2} = 0.125$ b) $1 - 0.8^2 = 0.36$

6. $\frac{3\,600 - 2 \cdot \frac{50 \cdot 50}{2}}{3\,600} = \frac{1\,100}{3\,600} = \frac{11}{36} \approx 0.306$

7. $\frac{3\,600 - \frac{40 \cdot 40}{2} - \frac{50 \cdot 50}{2}}{3\,600} = \frac{1\,550}{3\,600} \approx 0.431$

8. $\frac{3\,600 - \frac{45 \cdot 45}{2} - \frac{60 \cdot 60}{2}}{3\,600} = \frac{787.5}{3\,600} \approx 0.219$

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