

This handout looks at percent increase and percent decrease: how to compute them quickly using decimals, and how to work backwards when we know the result of an increase or decrease but not the original amount.

Example 1. a) Increase 300 by 20%. b) Find 120% of 300.

Solution: a) To increase 300 by 20%, we add 20% of 300 to 300 itself.

$$\begin{aligned} 300 + 20\% \text{ of } 300 &= 300 + \frac{20}{100} \cdot 300 \\ &= 300 + 60 = 360 \end{aligned}$$

So increasing 300 by 20% gives 360.

b) To find 120% of 300, we multiply.

$$120\% \text{ of } 300 = \frac{120}{100} \cdot 300 = 1.2 \cdot 300 = 360$$

So 120% of 300 is 360.

Example 2. a) Decrease 300 by 15%. b) Find 85% of 300.

Solution: a) To decrease 300 by 15%, we subtract 15% of 300 from 300 itself.

$$\begin{aligned} 300 - 15\% \text{ of } 300 &= 300 - \frac{15}{100} \cdot 300 \\ &= 300 - 45 = 255 \end{aligned}$$

So decreasing 300 by 15% gives 255.

b) To find 85% of 300, we multiply.

$$85\% \text{ of } 300 = \frac{85}{100} \cdot 300 = 0.85 \cdot 300 = 255$$

So 85% of 300 is 255.

Notice that in both examples, part a) and part b) gave us the very same answer. This is not a coincidence. When we increase a quantity by 20%, we end up with the original 100% of it, plus another 20% of it – that is, $100\% + 20\% = 120\%$ of the original amount. So increasing something by 20% is the very same computation as finding 120% of it. Similarly, when we decrease a quantity by 15%, we are left with $100\% - 15\% = 85\%$ of the original amount. So decreasing something by 15% is the same as finding 85% of it.

In computations, the easiest way to handle percents is to convert them into decimals right away, and to think of a percent increase or decrease as multiplication by a single decimal number. To increase a quantity Q by 20%, we compute

$$Q + 20\% \text{ of } Q = 100\% \text{ of } Q + 20\% \text{ of } Q = 120\% \text{ of } Q = 1.2Q.$$

So increasing Q by 20% is quickly computed as $1.2Q$. In the same way, decreasing Q by 15% leaves us with $100\% - 15\% = 85\%$ of Q , which is $0.85Q$. We will use this idea for the remainder of this handout: a percent increase or decrease turns into multiplication by a single decimal number, $(1 + r)$ for an increase of r (as a decimal), or $(1 - r)$ for a decrease of r .

This idea is also very useful when we know the result of an increase or decrease, but need to find the original amount.

Example 3. Sally got a 5% raise at work. Now she makes \$3570 a month. How much was she making before the raise?

Solution: Let us denote Sally's salary before the raise by x . A 5% raise means her new salary is 105% of x , that is, $1.05x$. This new salary is \$3570, so our equation is $1.05x = 3570$.

$$1.05x = 3570 \quad \text{divide by 1.05}$$

$$x = 3400$$

We check: a 5% raise on \$3400 is $1.05(3400) = 3570$, which matches her new salary. So Sally was making \$3400 a month before the raise.

Example 4. A coat went on a 20% off sale. The sale price is \$96. What was the original price?

Solution: Let us denote the original price by x . A 20% discount means the sale price is 80% of the original price, that is, $0.8x$. This sale price is \$96, so our equation is $0.8x = 96$.

$$0.8x = 96 \quad \text{divide by 0.8}$$

$$x = 120$$

We check: 20% off of \$120 is $0.8(120) = 96$, which matches the sale price. So the original price was \$120.

We can also work in the opposite direction: given the decimal multiplier, figure out what percent increase (or decrease) it represents.

Example 5. For each of the given multipliers, state what percent increase (or decrease) of Q it represents.

- a) $1.02Q$ b) $1.12Q$ c) $1.2Q$ d) $5Q$

Solution: We compare each multiplier to $1Q$, which represents 100% of Q , that is, no change at all.

a) $1.02Q$ is 2% more than Q , since $1.02 = 1 + 0.02$. So this is a 2% increase.

b) $1.12Q$ is 12% more than Q , since $1.12 = 1 + 0.12$. So this is a 12% increase.

c) $1.2Q$ is 20% more than Q , since $1.2 = 1 + 0.20$. So this is a 20% increase.

d) $5Q$ is 400% more than Q , since $5 = 1 + 4 = 100\% + 400\%$. So this is a 400% increase. (Even though $5Q$ is five *times* as large as Q , it represents a 400% *increase*, not a 500% increase – Q itself is always the starting 100%.)

Finally, what happens if a quantity is changed by one percent, and then the result is changed by another percent? Since each percent change is just multiplication by a decimal, we can combine two changes into a single change by multiplying the two decimal multipliers together.

Example 6. A quantity Q is increased by 20%, and then the result is increased by 30%. Express the combined change as a single percent change.

Solution: Increasing Q by 20% gives $1.2Q$. Increasing that result by 30% means multiplying it by 1.3, which gives

$$1.3(1.2Q) = 1.56Q.$$

Since $1.56 = 1 + 0.56$, the combined effect of the two increases is the same as a single 56% increase. Notice that the answer is *not* $20\% + 30\% = 50\%$: the second increase is applied to the already-larger amount $1.2Q$, not to the original Q , so the two percent changes do not simply add.



Sample Problems

For problems 1-4, either increase/decrease the given number by the given percent, or find the given percent of the number, as indicated.

1. Increase 250 by 40%.
2. Find 140% of 250.
3. Decrease 250 by 12%.
4. Find 88% of 250.

For problem 5, express the given combined change as a single percent change.

5. A quantity is increased by 40%, and then the result is decreased by 25%.

Solve each of the following application problems.

6. Tom's rent increased by 8% this year. His new rent is \$972 a month. What was his rent before the increase?
7. A shirt is on sale for 30% off. The sale price is \$42. What was the original price?



Practice Problems

For problems 1-6, either increase/decrease the given number by the given percent, or find the given percent of the number, as indicated.

1. Increase 180 by 25%.
2. Find 125% of 180.
3. Decrease 400 by 35%.
4. Find 65% of 400.
5. Increase 90 by 150%.
6. Decrease 500 by 8%.

Solve each of the following application problems.

7. Maria's salary increased by 4%. She now makes \$3120 a month. What was her salary before the raise?
8. A laptop is on sale for 15% off. The sale price is \$680. What was the original price?
9. The population of a town increased by 6% this year, to 21,200 people. What was the population last year?

For problems 10-13, state what percent increase (or decrease) of Q each multiplier represents.

10. $1.15Q$
11. $0.75Q$
12. $2Q$
13. $0.4Q$

For problems 14-15, express the given combined change as a single percent change.

14. A number is increased by 10%, and then the result is increased by 10% again.
15. A price is increased by 50%, and then the result is decreased by 50%.



Answers

Sample Problems

1. 350 2. 350 3. 220 4. 220
5. 5% increase.
6. Tom's rent was \$900 before the increase. 7. The original price was \$60.

Practice Problems

1. 225 2. 225 3. 260 4. 260 5. 225 6. 460
7. Maria's salary was \$3000 before the raise. 8. The original price was \$800. 9. The population last year was 20,000.
10. 15% increase 11. 25% decrease 12. 100% increase 13. 60% decrease
14. 21% increase. 15. 25% decrease.

Sample Problems



Solutions

For problems 1-4, either increase/decrease the given number by the given percent, or find the given percent of the number, as indicated.

1. Increase 250 by 40%.

Solution: Increasing by 40% means multiplying by $1 + 0.40 = 1.4$.

$$250 \times 1.4 = 350$$

Thus increasing 250 by 40% gives 350.

2. Find 140% of 250.

Solution: 140% as a decimal is 1.4.

$$250 \times 1.4 = 350$$

Thus 140% of 250 is 350. (Notice this is the same computation, and the same answer, as problem 1 – increasing a number by 40% is the same as finding 140% of it.)

3. Decrease 250 by 12%.

Solution: Decreasing by 12% means multiplying by $1 - 0.12 = 0.88$.

$$250 \times 0.88 = 220$$

Thus decreasing 250 by 12% gives 220.

4. Find 88% of 250.

Solution: 88% as a decimal is 0.88.

$$250 \times 0.88 = 220$$

Thus 88% of 250 is 220.

5. A quantity is increased by 40%, and then the result is decreased by 25%.

Solution: Increasing by 40% means multiplying by 1.4, and then decreasing that result by 25% means multiplying by 0.75. The combined multiplier is

$$1.4 \times 0.75 = 1.05.$$

Since $1.05 = 1 + 0.05$, the combined change is a single 5% increase. Even though the second step is a decrease, the overall combined effect is still an increase, since the 40% increase was applied to a larger starting amount than the 25% decrease was.

6. Tom's rent increased by 8% this year. His new rent is \$972 a month. What was his rent before the increase?

Solution: Let us denote Tom's rent before the increase by x . An 8% increase means his new rent is $1.08x$.

$$1.08x = 972 \quad \text{divide by 1.08}$$

$$x = 900$$

We check: an 8% increase on \$900 is $1.08(900) = 972$, which matches. So Tom's rent was \$900 before the increase.

7. A shirt is on sale for 30% off. The sale price is \$42. What was the original price?

Solution: Let us denote the original price by x . A 30% discount means the sale price is $0.7x$.

$$0.7x = 42 \quad \text{divide by 0.7}$$

$$x = 60$$

We check: 30% off of \$60 is $0.7(60) = 42$, which matches the sale price. So the original price was \$60.