

Suppose that we are told that we will receive 20 000 dollars exactly 5 years from today. For us, today, this guarantee is not worth 20 000 dollars, because, over time, prices go up and money loses its purchase power. Consider the Macy's menu from 1917, shown on the picture. Just a bit more than hundred years ago, a glass of ginger ale at a diner cost 19 cents. The same drink at the same place would cost us 5-10 times as much today. This phenomenon, present in good and bad economies, is called **inflation**.



Suppose we have the guarantee that we will receive 20 000 dollars exactly 5 years from today, but we need money right now. Could we sell this guarantee? Clearly not for 20 000 dollars. On the other hand, it feels like it would be a great bargain if we could buy this guarantee for a lower amount, say 5000 dollars. So then what is the current value of this guarantee? While this question sounds difficult, it becomes quite easy if we re-phrase it cleverly.

Let us consider this question from the other side. If we wanted to buy such a guarantee, how much are we willing to pay for it? Suppose that we have a bank account whose interest rate matches exactly the inflation. We would not want to buy this guarantee if leaving our money in the bank would result in just as much profit as this purchase. So, here is an easier question.

Example 1. Assume an annual compound interest rate of 4%, compounded annually. How much money do we need to deposit into this account so that five years from today the account would increase to contain 20 000 dollars?

Solution: This time the final amount is given and the principal is what we need to find. If we denote the principal by x , then we can write the equation using $r = 0.04$, $t = 5$ and final amount $A = 20\,000$.

$$20\,000 = x \cdot 1.04^5$$

To solve this equation for x , we need to divide both sides of this equation by whatever is multiplying x , in this case, 1.04^5

$$x = \frac{\$20\,000}{1.04^5} \approx \$16\,438.54$$

So, we can say that under an assumed annual inflation rate of 4%, the present value of 20 000 dollars five years from today is 16 438.54 dollars.

As it turns out, computing the present value of a future event is not that difficult. It is probably a much more difficult problem to reliably predict the inflation rate for five years in the future. That is where the real hard work lies. Once we are given the annual inflation rate (sometimes still called interest rate), we just state our usual formula and then solve the resulting equation for the principal. Even though the compound interest formulas can be complicated, they are always easy to solve for the principal, as we go from

$$A = P \left(1 + \frac{r}{n}\right)^{nt} \quad \text{to} \quad P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

where A is the final amount in the account, P is the principal, r the annual interest rate as a decimal, t the number of years, and n the number of times interest is credited in a year. The formula is not that much more difficult in case of simple interest:

$$A = P + P r t = P(1 + r t) \quad \text{becomes} \quad P = \frac{A}{1 + r t}$$

Example 2. Your rich uncle has promised to give you 50 000 dollars, once you graduate from college. Let's assume that that will happen in five years. What is the present value of this gift today, assuming an annual

- a) 6% simple interest rate
- b) 6% compound interest rate, compounded annually
- c) 6% compound interest rate, compounded semi-annually
- d) 6% compound interest rate, compounded monthly
- e) 6% compound interest rate, compounded daily?

Solution: a) We place x into a bank account with an annual simple interest rate of 6%. What is x if after five years, the account increases to contain \$ 50 000? We set up and solve the equation

$$\begin{aligned} x(1 + 5 \cdot 0.06) &= 50\,000 \\ x \cdot 1.3 &= 50\,000 \\ x &= \frac{50\,000}{1.3} \approx 38\,461.54 \end{aligned}$$

So the present value of this gift is \$38 461.54.

b) We place x into a bank account with an annual compound interest rate of 6%, compounded annually. What is x if after five years, the account increases to contain \$ 50 000? We set up and solve the equation

$$\begin{aligned} x \cdot 1.06^5 &= 50\,000 \\ x &= \frac{50\,000}{1.06^5} \approx 37\,362.91 \end{aligned}$$

So the present value of this gift is \$37 362.91.

c) We place x into a bank account with an annual compound interest rate of 6%, compounded semi-annually. What is x if after five years, the account increases to contain \$ 50 000? We set up and solve the equation. With semi-annual compounding, we get a 3% increase twice a year.

$$\begin{aligned} x \cdot 1.03^{10} &= 50\,000 \\ x &= \frac{50\,000}{1.03^{10}} \approx 37\,204.70 \end{aligned}$$

So the present value of this gift is \$37 204.70. $\frac{50\,000}{1.005^{60}} = 37068.609812217$

d) We place x into a bank account with an annual compound interest rate of 6%, compounded monthly. What is x if after five years, the account increases to contain \$ 50 000? We set up and solve the equation. With semi-annual compounding, we get a 0.5% increase twelve times a year.

$$\begin{aligned} x \cdot 1.005^{60} &= 50\,000 \\ x &= \frac{50\,000}{1.005^{60}} \approx 37\,068.61 \end{aligned}$$

So the present value of this gift is \$37 068.61.

- e) We place x into a bank account with an annual compound interest rate of 6%, compounded daily. What is x if after five years, the account increases to contain \$ 50 000? We set up and solve the equation With semi-annual compounding, we get a $\frac{6\%}{360}$ increase once a day..

$$\begin{aligned}
 x \left(1 + \frac{0.06}{360} \right)^{5 \cdot 360} &= 50\,000 \\
 x \left(1 + \frac{0.06}{360} \right)^{1800} &= 50\,000 \\
 x &= \frac{50\,000}{\left(1 + \frac{0.06}{360} \right)^{1800}} \approx 37\,041.84
 \end{aligned}$$

So the present value of this gift is \$37 041.84.

Example 3. You will receive \$5000 today, then \$5000 a year from today, and then \$5000 two years from today. What is the present value of these three payments today, assuming an annual 4% compound interest rate,

- a) compounded annually b) compounded daily

Solution: a) There are two things to notice about this problem. First, instead of one difficult problem, we have three easy problems. Second, that the trickiest part is entering computations into the calculator without losing precision.

\$5000 today is worth \$5000.

\$5000 a year from today:

$$\begin{aligned}
 x \cdot 1.04 &= 5000 \\
 x &= \frac{5000}{1.04} \approx 4807.69
 \end{aligned}$$

and \$5000 two years from today:

$$\begin{aligned}
 x \cdot 1.04^2 &= 5000 \\
 x &= \frac{5000}{1.04^2} \approx 4622.78
 \end{aligned}$$

Our answer is most precise if we don't round before adding the three results. So, instead of entering $5000 + 4807.69 + 4622.78$, we might get a more precise result when entering $5000 + \frac{5000}{1.04} + \frac{5000}{1.04^2}$ into the calculator. The difference between the results is .0003373 - not enough to affect the result rounded to the nearest penny.

- b) \$5000 today is worth \$5000.

\$5000 a year from today:

$$\begin{aligned}
 x \left(1 + \frac{0.04}{360} \right)^{360} &= 5000 \\
 x &= \frac{5000}{\left(1 + \frac{0.04}{360} \right)^{360}} \approx 4803.96
 \end{aligned}$$

and \$5000 two years from today:

$$\begin{aligned}x \left(1 + \frac{0.04}{360}\right)^{2 \cdot 360} &= 5000 \\x \left(1 + \frac{0.04}{360}\right)^{720} &= 5000 \\x &= \frac{5000}{\left(1 + \frac{0.04}{360}\right)^{720}} \approx 4615.60\end{aligned}$$

Our answer is most precise if we don't round before adding the three results. So, instead of entering $5000 + 4803.96 + 4615.60$ we might get a more precise result when entering

$$5000 + \frac{5000}{\left(1 + \frac{0.04}{360}\right)^{360}} + \frac{5000}{\left(1 + \frac{0.04}{360}\right)^{720}}$$

into the calculator. Both computations will result in \$14419.56 for the present value.

We have a calculator that can handle the entire line $5000 + \frac{5000}{\left(1 + \frac{0.04}{360}\right)^{360}} + \frac{5000}{\left(1 + \frac{0.04}{360}\right)^{2 \cdot 360}}$ However, this is a very long line to enter again and again in case we make a mistake. So, how can we make life easier without losing precision?

One way to get around this is to enter $\frac{5000}{\left(1 + \frac{0.04}{360}\right)^{360}}$ and $\frac{5000}{\left(1 + \frac{0.04}{360}\right)^{2 \cdot 360}}$ separately, writing down the numbers, but with more precision than to a penny (4-5 digits is sufficient). This way we can clear the calculator and re-enter the partial results. There is an easier way, however. Our calculator can store up to five numbers in its memory.

We can enter $\frac{5000}{\left(1 + \frac{0.04}{360}\right)^{360}}$ in its entirety as is -do not clear the calculator after the division. Then we can store this value by pressing the STO button STO is for store. When we push the store button, we can toggle between values of A, B, C, D, and E. If there are already values stored, they are shown. We use the horizontal arrows to toggle between the place holders and select by pushing the = button. Then we can clear the calculator and enter another complex expression, say $\frac{5000}{\left(1 + \frac{0.04}{360}\right)^{2 \cdot 360}}$. Make sure that the expression $2 \cdot 360$ is enclosed in a pair of parentheses. We can store this result in B.

We can recall these values using the same button but with pushing the 2nd button first. We make our selection using the arrows and the = button. The only annoying thing is that once these numbers are recalled, the last few digits are shown, not the beginning of the number.