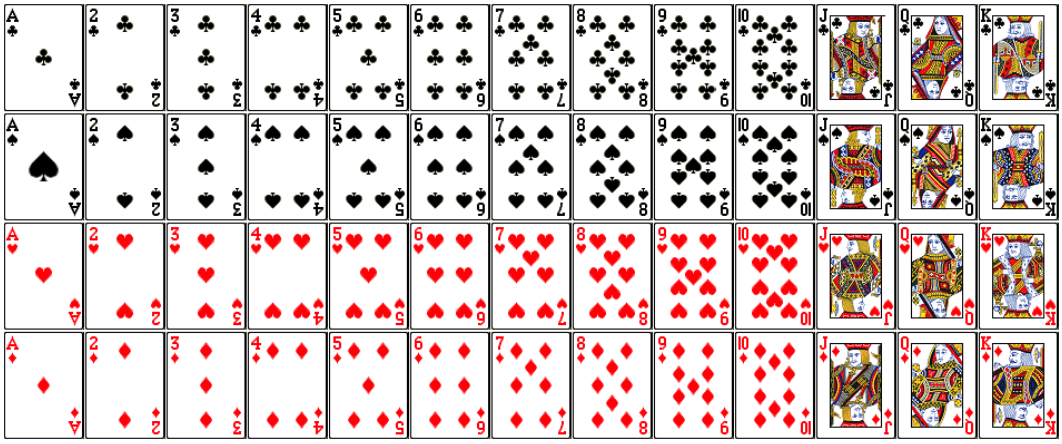
Recall that when all outcomes of an experiment are equally likely, the probability of an event E is $P(E) = \dfrac{n(E)}{n(S)}$, the number of outcomes in E divided by the number of outcomes in the sample space S. Many events are built from simpler ones with the word "or." In this handout we learn how to find the probability of such an event without listing every outcome.

Definition. Let E and F be events of the same experiment. The event E **or** F occurs when *at least one* of E and F occurs. The event E **and** F occurs when *both* E and F occur.

In the language of sets, "E or F" is the union $E \cup F$. We form $E \cup F$ by putting the outcomes of E and F together, so a single outcome belongs to $E \cup F$ when it is in E or in F (or in both). Similarly, "E and F" is the intersection $E \cap F$: a single outcome belongs to $E \cap F$ when it is in both E and F. Before we state the rule, let us see what can go wrong when we simply add.

A standard deck has 52 cards: four suits (clubs, spades, hearts, diamonds), each with thirteen ranks. The jacks, queens, and kings are the 12 face cards.



Example 1. One card is drawn at random from a standard deck. Find the probability that it is a heart or a face card.

Solution. There are 13 hearts and 12 face cards. It is tempting to say that $13 + 12 = 25$ cards are hearts or face cards, but three cards are *both*: the jack, queen, and king of hearts. These three cards were counted twice, once among the hearts and once among the face cards. To count each card only once, we subtract them:

$$n(\text{heart or face card}) = 13 + 12 - 3 = 22 \qquad \text{so} \qquad P(\text{heart or face card}) = \frac{22}{52} = \frac{11}{26}.$$

In terms of probabilities, this is

$$\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} \qquad \text{that is,} \qquad P(\text{heart}) + P(\text{face card}) - P(\text{heart and face card}).$$

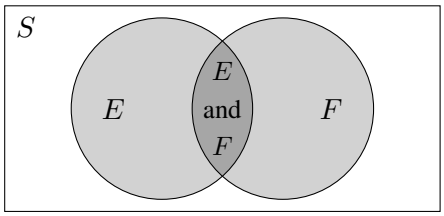
© Hidegkuti, 2026

This is not a coincidence. The same reasoning works for any two events.

The OR Rule (Addition Rule). For any two events E and F,

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F).$$

The Venn diagram shows why we subtract. The outcomes in the overlap are in both E and F, so $P(E) + P(F)$ counts them twice. Subtracting $P(E \text{ and } F)$ counts them once.



Sometimes two events cannot happen at the same time. For example, a single die cannot show both a 2 and a 5.

Definition. Two events E and F are **mutually exclusive** if they cannot both occur, that is, if "E and F" is the impossible event. For mutually exclusive events, $P(E \text{ and } F) = 0$, so the OR rule becomes

$$P(E \text{ or } F) = P(E) + P(F).$$

Example 2. A fair die is rolled. Find the probability that the number rolled is

a) even or a 5, b) even or greater than 3.

Solution. a) The even numbers are 2, 4, 6, and 5 is odd, so the two events are mutually exclusive.

$$P(\text{even or } 5) = P(\text{even}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

b) The even numbers are 2, 4, 6, and the numbers greater than 3 are 4, 5, 6. The numbers 4 and 6 belong to both events.

$$P(\text{even or greater than } 3) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

We can check this answer by listing the outcomes in the event: 2, 4, 5, 6. There are 4 of them, so the probability is $\frac{4}{6}$.

© Hidegkuti, 2026

Example 3. A fair die is tossed twice. Find the probability that

a) the sum is 7 or both tosses show the same number (doubles),

b) the sum is 8 or both tosses show the same number.

Solution. There are 36 equally likely outcomes. Six of them are doubles: $(1,1)$, $(2,2)$, $\ldots$, $(6,6)$.

a) Six outcomes have sum 7: $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$, $(6,1)$. None of these are doubles (doubles always have an even sum), so the events are mutually exclusive.

$$P(\text{sum is 7 or doubles}) = \frac{6}{36} + \frac{6}{36} = \frac{12}{36} = \frac{1}{3}$$

b) Five outcomes have sum 8: $(2,6)$, $(3,5)$, $(4,4)$, $(5,3)$, $(6,2)$. One of them, $(4,4)$, is also a double.

$$P(\text{sum is 8 or doubles}) = \frac{5}{36} + \frac{6}{36} - \frac{1}{36} = \frac{10}{36} = \frac{5}{18}$$

Example 4. A college surveyed 200 students about whether they have a job. The results are shown in the table.

	works	does not work	total
freshman	45	75	120
sophomore	50	30	80
total	95	105	200

One of these students is chosen at random. Find the probability that the student is a freshman or works.

Solution. There are 120 freshmen and 95 students who work. The 45 working freshmen belong to both groups.

$$P(\text{freshman or works}) = \frac{120}{200} + \frac{95}{200} - \frac{45}{200} = \frac{170}{200} = \frac{17}{20}$$

Be careful. If we forget to subtract the overlap, we may even get a "probability" greater than 1. For example, when one card is drawn, there are 26 red cards and 48 cards that are not aces, and $\frac{26}{52} + \frac{48}{52} = \frac{74}{52} > 1$. The 24 red cards that are not aces were counted twice. The correct answer is

$$P(\text{red or not an ace}) = \frac{26}{52} + \frac{48}{52} - \frac{24}{52} = \frac{50}{52} = \frac{25}{26}.$$

© Hidegkuti, 2026

The Complement Rule

The **complement** of an event E, written "not E", consists of all outcomes of the sample space that are *not* in E. An event and its complement are mutually exclusive, and together they contain every outcome, so $P(E) + P(\text{not } E) = 1$.

The Complement Rule. For any event E,

$$P(\text{not } E) = 1 - P(E).$$

The complement rule is useful when an event has many outcomes but its complement has only a few.

Example 5. A fair die is tossed twice. Find the probability that the sum is at least 4.

Solution. The complement of "the sum is at least 4" is "the sum is 2 or 3." Only three outcomes have such a sum: $(1, 1)$, $(1, 2)$, $(2, 1)$. So

$$P(\text{sum is at least } 4) = 1 - P(\text{sum is 2 or 3}) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12}.$$

Example 6. In the survey above, find the probability that a randomly chosen student is a freshman or works, using the complement rule.

Solution. A student who is *not* a freshman and does *not* work is a sophomore who does not work. There are 30 such students, so

$$P(\text{freshman or works}) = 1 - \frac{30}{200} = \frac{170}{200} = \frac{17}{20}.$$

This agrees with the answer we found with the OR rule.

© Hidegkuti, 2026



Practice Problems

1. A fair die is rolled. Find the probability that the number rolled is odd or less than 3.

2. One card is drawn at random from a standard deck of 52 cards. Find the probability that it is

   a) a queen or a club, b) red or an ace, c) a king or a queen.

3. A fair die is tossed twice. Find the probability that

   a) the sum is 6 or 11, b) the sum is 10 or the tosses are doubles, c) at least one toss is a 6.

4. Suppose that $P(E) = 0.45$, $P(F) = 0.30$, and $P(E \text{ and } F) = 0.12$. Find

   a) $P(E \text{ or } F)$, b) $P(\text{not } E)$.

5. Suppose that E and F are mutually exclusive, $P(E) = 0.2$, and $P(F) = 0.5$. Find

   a) $P(E \text{ or } F)$, b) $P(\text{not } (E \text{ or } F))$.

6. A coffee shop asked 300 adults whether they drink coffee every day.

	drinks coffee	does not	total
under 30	60	90	150
30 or older	100	50	150
total	160	140	300

   One of these adults is chosen at random. Find the probability that the person

   a) drinks coffee or is under 30, b) does not drink coffee, c) drinks coffee and is under 30.

7. A bag contains 4 red, 6 blue, and 5 green marbles. One marble is drawn at random. Find the probability that it is

   a) red or green, b) not red.

8. A student says: "There are 13 hearts and 4 kings, so the probability of drawing a heart or a king is $\frac{13}{52} + \frac{4}{52} = \frac{17}{52}$." What is wrong? Find the correct probability.

9. A fair coin is tossed three times. Use the complement rule to find the probability of getting at least one head.

© Hidegkuti, 2026



Answers

1. The odd numbers are 1, 3, 5, the numbers less than 3 are 1 and 2, and 1 is in both events: $\frac{3}{6}+\frac{2}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

2. a) $\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$ b) $\frac{26}{52}+\frac{4}{52}-\frac{2}{52}=\frac{28}{52}=\frac{7}{13}$
   c) The events are mutually exclusive: $\frac{4}{52}+\frac{4}{52}=\frac{8}{52}=\frac{2}{13}$

3. a) The events are mutually exclusive: $\frac{5}{36}+\frac{2}{36}=\frac{7}{36}$ b) $\frac{3}{36}+\frac{6}{36}-\frac{1}{36}=\frac{8}{36}=\frac{2}{9}$
   c) The complement is "no 6 on either toss," which has $5\cdot 5=25$ outcomes: $1-\frac{25}{36}=\frac{11}{36}$

4. a) $0.45+0.30-0.12=0.63$ b) $1-0.45=0.55$

5. a) $0.2+0.5=0.7$ b) $1-0.7=0.3$

6. a) $\frac{160}{300}+\frac{150}{300}-\frac{60}{300}=\frac{250}{300}=\frac{5}{6}$ b) $\frac{140}{300}=\frac{7}{15}$ c) $\frac{60}{300}=\frac{1}{5}$

7. There are 15 marbles. a) $\frac{4}{15}+\frac{5}{15}=\frac{9}{15}=\frac{3}{5}$ b) $1-\frac{4}{15}=\frac{11}{15}$

8. The king of hearts was counted twice, once as a heart and once as a king. The correct probability is $\frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$.

9. The complement of "at least one head" is "no heads," which is only the outcome TTT: $1-\frac{1}{8}=\frac{7}{8}$

For more documents like this, visit our page at https://teaching.martahidegkuti.com and click on Lecture Notes. E-mail questions or comments to mhidegkuti@ccc.edu.

© Hidegkuti, 2026