We have solved quadratic equations by factoring. Unfortunately, many quadratic equations can not be factored by trial and error. For example, the equation $x^2 - x - 1 = 0$ comes up naturally in several problems below, and there are no two integers whose product is -1 and whose sum is -1. The quadratic formula solves every quadratic equation, whether it can be factored or not.

Theorem: (The quadratic formula) Suppose that a, b, and c are real numbers, where $a \neq 0$. Consider the quadratic equation

$$ax^2 + bx + c = 0$$

If $b^2 - 4ac$ is negative, the equation has no real solution. If $b^2 - 4ac$ is non-negative, then the solution(s) of the quadratic equation are x_1 and x_2 and they can be computed by the formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ is called the **discriminant**. When the discriminant is positive, the equation has two different real solutions. When the discriminant is zero, the equation has exactly one real solution. When the discriminant is negative, the equation has no real solution.

Example 1. Solve $2x^2 - 9x = 5$ over the real numbers.

Solution: We first reduce one side to zero: $2x^2 - 9x - 5 = 0$. Now we can apply the quadratic formula.

$a = 2$, $b = -9$, and $c = -5$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \cdot 2 (-5)}}{2 \cdot 2} = \frac{9 \pm \sqrt{81 - (-40)}}{4} = \frac{9 \pm \sqrt{81 + 40}}{4}$$

$$= \frac{9 \pm \sqrt{121}}{4} = \frac{9 \pm 11}{4} = \begin{cases} \frac{9 + 11}{4} = \frac{20}{4} = 5 \\ \frac{9 - 11}{4} = \frac{-2}{4} = -\frac{1}{2} \end{cases}$$

So this equation has two real solutions, $\boxed{5 \text{ and } -\frac{1}{2}}$. We check: if $x = 5$, then

$$\text{LHS} = 2(5)^2 - 9(5) = 50 - 45 = 5 = \text{RHS}$$

and if $x = -\frac{1}{2}$, then $\text{LHS} = 2\left(-\frac{1}{2}\right)^2 - 9\left(-\frac{1}{2}\right) = 2 \cdot \frac{1}{4} + \frac{9}{2} = \frac{1}{2} + \frac{9}{2} = \frac{10}{2} = 5 = \text{RHS}$ and so both solutions are correct.

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Example 2. Solve $x^2 - 7x = -10 - x$ over the real numbers.

Solution: We first reduce one side to zero.

$$\begin{aligned} x^2 - 7x &= -10 - x && \text{add } x \\ x^2 - 6x &= -10 && \text{add } 10 \\ x^2 - 6x + 10 &= 0 \end{aligned}$$

Now we can apply the quadratic formula with values $a = 1$, $b = -6$, and $c = 10$.

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 - 40}}{2} = \frac{6 \pm \sqrt{-4}}{2} = \text{undefined}$$

We now see that the discriminant is negative. This means that the equation has $\boxed{\text{no real solutions}}$.

Example 3. Solve $2x^2 - 10x + 26 = 6x$ over the real numbers.

Solution: Notice that each term is even, and so we can divide both sides by 2.

$$\begin{aligned} 2x^2 - 10x + 26 &= 6x && \text{subtract } 6x \\ 2x^2 - 16x + 26 &= 0 \\ 2\left(x^2 - 8x + 13\right) &= 0 \\ x^2 - 8x + 13 &= 0 \end{aligned}$$

Now we can apply the quadratic formula with values $a = 1$, $b = -8$, and $c = 13$.

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 13}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 52}}{2} = \frac{8 \pm \sqrt{12}}{2}$$

However, this is not the most simplified form of the solution. Recall that $\sqrt{ab} = \sqrt{a}\sqrt{b}$ for all non-negative real numbers a and b. Using this rule, we obtain that $\sqrt{12} = 2\sqrt{3}$ as follows.

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

The solution can be further simplified:

$$x_{1,2} = \frac{8 \pm \sqrt{12}}{2} = \frac{8 \pm 2\sqrt{3}}{2} = \frac{2\left(4 \pm \sqrt{3}\right)}{2} = 4 \pm \sqrt{3}$$

This of course means that $\boxed{x_1 = 4 + \sqrt{3} \text{ and } x_2 = 4 - \sqrt{3}}$.

We check: if $x = 4 + \sqrt{3}$, then

$$\begin{aligned} \text{LHS} &= 2\left(4 + \sqrt{3}\right)^2 - 10\left(4 + \sqrt{3}\right) + 26 = 2\left(16 + 3 + 8\sqrt{3}\right) - 10\left(4 + \sqrt{3}\right) + 26 \\ &= 2\left(19 + 8\sqrt{3}\right) - 10\left(4 + \sqrt{3}\right) + 26 = 38 + 16\sqrt{3} - 40 - 10\sqrt{3} + 26 = 24 + 6\sqrt{3} \\ \text{RHS} &= 6\left(4 + \sqrt{3}\right) = 24 + 6\sqrt{3} \end{aligned}$$

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and so $x_1 = 4 + \sqrt{3}$ works. And if $x = 4 - \sqrt{3}$, then

$$\begin{aligned} \text{LHS} &= 2\left(4-\sqrt{3}\right)^2 - 10\left(4-\sqrt{3}\right) + 26 = 2\left(16+3-8\sqrt{3}\right) - 10\left(4-\sqrt{3}\right) + 26 \\ &= 2\left(19-8\sqrt{3}\right) - 10\left(4-\sqrt{3}\right) + 26 = 38 - 16\sqrt{3} - 40 + 10\sqrt{3} + 26 = 24 - 6\sqrt{3} \\ \text{RHS} &= 6\left(4-\sqrt{3}\right) = 24 - 6\sqrt{3} \end{aligned}$$

and so $x_2 = 4 - \sqrt{3}$ also works.

Example 4. Solve $9x^2 + 1 = 6x$ over the real numbers.

Solution:

$$\begin{aligned} 9x^2 + 1 &= 6x \\ 9x^2 - 6x + 1 &= 0 \end{aligned}$$

We apply the quadratic formula with values $a = 9$, $b = -6$, and $c = 1$.

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 9 \cdot 1}}{2 \cdot 9} = \frac{6 \pm \sqrt{36-36}}{18} = \frac{6 \pm \sqrt{0}}{18} = \frac{6}{18} = \frac{1}{3}$$

So this equation has only one real solution: $x = \frac{1}{3}$. We check: if $x = \frac{1}{3}$, then

$$\text{LHS} = 9\left(\frac{1}{3}\right)^2 + 1 = 9 \cdot \frac{1}{9} + 1 = 2 \text{ and RHS} = 6 \cdot \frac{1}{3} = \frac{6}{3} = 2$$

And so our solution is correct.

Example 5. Find all real numbers with the following property: the number is exactly 3 greater than its own reciprocal.

Solution: Let x denote this number. Then the equation expressing the stated property is: $x = \frac{1}{x} + 3$. We will solve this equation for x. We clear the denominator by multiplying both sides by x. We are then left with a quadratic equation we will solve using the quadratic formula.

$$\begin{aligned} x &= \frac{1}{x} + 3 \qquad \text{multiply by } x \\ x^2 &= 1 + 3x \\ x^2 - 3x - 1 &= 0 \end{aligned}$$

We apply the quadratic formula with values $a = 1$, $b = -3$, and $c = -1$.

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

So there are two such numbers. We check: if $x = \frac{3+\sqrt{13}}{2}$, then its reciprocal, $\frac{1}{x} = \frac{2}{\sqrt{13}+3}$. We rationalize the denominator:

$$\frac{2}{\sqrt{13}+3} = \frac{2}{\sqrt{13}+3} \cdot \frac{\sqrt{13}-3}{\sqrt{13}-3} = \frac{2\left(\sqrt{13}-3\right)}{4} = \frac{\sqrt{13}-3}{2}$$

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Clearly, $\frac{1}{x} = \frac{\sqrt{13}-3}{2}$ is exactly 3 less than $x = \frac{\sqrt{13}+3}{2}$. Checking the other number goes similarly. Therefore, our solution is correct.

Example 6. In each of the given equations, use the discriminant to determine the number of real solutions without solving the equation.

a) $5x^2 - 3x + 1 = 0$

b) $-2x^2 + 7x + 2 = 0$

c) $\frac{1}{2}x^2 - x + 1 = 0$

d) $8x^2 + \frac{1}{2} = 4x$

e) $3x^2 - 12x + 87 = 0$

f) $13x + 3x^2 = 56$

g) $9x^2 - 30x + 25 = 0$

Solution: We will compute the value of the discriminant, $D = b^2 - 4ac$. If the discriminant is positive, there are two real solutions. If it is zero, there is exactly one real solution. If it is negative, there is no real solution.

a) $5x^2 - 3x + 1 = 0$. In this case, $a = 5$, $b = -3$, and $c = 1$.

$$D = b^2 - 4ac = (-3)^2 - 4 \cdot 5 \cdot 1 = 9 - 20 = -11$$

Since the discriminant is negative, there is no real solution for this equation.

b) $-2x^2 + 7x + 2 = 0$. In this case, $a = -2$, $b = 7$, and $c = 2$.

$$D = b^2 - 4ac = 7^2 - 4\,(-2)\,2 = 49 - (-16) = 65$$

Since the discriminant is positive, there are two real solutions for this equation.

c) $\frac{1}{2}x^2 - x + 1 = 0$. In this case, $a = \frac{1}{2}$, $b = -1$, and $c = 1$.

$$D = b^2 - 4ac = (-1)^2 - 4 \cdot \frac{1}{2} \cdot 1 = 1 - 2 = -1$$

Since the discriminant is negative, there is no real solution for this equation.

d) $8x^2 + \frac{1}{2} = 4x$.

Solution: Caution! We can not apply the quadratic formula if the equation is in this form. Remember, one side must be zero.

$$\begin{aligned} 8x^2 + \frac{1}{2} &= 4x \qquad \text{subtract } 4x \\ 8x^2 - 4x + \frac{1}{2} &= 0 \end{aligned}$$

And so in this case, $a = 8$, $b = -4$, and $c = \frac{1}{2}$.

$$D = b^2 - 4ac = (-4)^2 - 4 \cdot 8 \cdot \frac{1}{2} = 16 - 16 = 0$$

Since the discriminant is zero, there is exactly one real solution for this equation.

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e) $3x^2 - 12x + 87 = 0$

Solution: At first it appears that $a = 3$, $b = -12$, and $c = 87$. We might notice that all of a, b, and c are divisible by 3. It might make things easier if we divide both sides of the equation by 3 before applying the quadratic formula.

$$\begin{aligned} 3x^2 - 12x + 87 &= 0 \\ 3\left(x^2 - 4x + 29\right) &= 0 \qquad \text{divide by 3} \\ x^2 - 4x + 29 &= 0 \end{aligned}$$

So now $a = 1$, $b = -4$, and $c = 29$.

$$D = b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 29 = 16 - 116 = -100$$

Since the discriminant is negative, there is no real solution for this equation.

f) $13x + 3x^2 = 56$

Solution: We first reduce one side to zero. $3x^2 + 13x - 56 = 0$. So $a = 3$, $b = 13$, and $c = -56$.

$$D = b^2 - 4ac = 13^2 - 4 \cdot 3\,(-56) = 169 - (-672) = 169 + 672 = 841$$

Since the discriminant is positive, there are two real solutions for this equation.

g) $9x^2 - 30x + 25 = 0$

Solution: $a = 9$, $b = -30$, and $c = 25$.

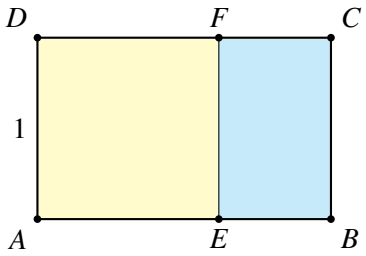
$$D = b^2 - 4ac = (-30)^2 - 4 \cdot 9 \cdot 25 = 900 - 900 = 0$$

Since the discriminant is zero, there is exactly one real solution for this equation.

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The rest of this handout shows how quadratic equations come up in geometry and in probability. In each case, the problem itself leads to a quadratic equation that can not be easily solved by factoring.

Example 7. (The golden rectangle) In the rectangle $ABCD$, the side AB is longer than the side AD, and $AD = 1$ unit. We cut off the square $AEFD$. The remaining rectangle $EBCF$ is similar to the rectangle $ABCD$. Find the exact value of the length of AB.



Solution: Let x denote the length of AB. The square has sides 1, so $AE = 1$ and $EB = x - 1$. In the rectangle $ABCD$, the longer side is $AB = x$ and the shorter side is $BC = 1$. In the rectangle $EBCF$, the sides are turned: the longer side is $BC = 1$ and the shorter side is $EB = x - 1$. The two rectangles are similar, so the ratio $\dfrac{\text{longer side}}{\text{shorter side}}$ is the same in both of them.

$$\begin{aligned} \frac{x}{1} &= \frac{1}{x-1} && \text{multiply by } x-1 \\ x(x-1) &= 1 \\ x^2 - x &= 1 && \text{distribute } x \\ x^2 - x - 1 &= 0 && \text{subtract 1} \end{aligned}$$

We apply the quadratic formula with values $a = 1$, $b = -1$, and $c = -1$.

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

The solution $\dfrac{1-\sqrt{5}}{2}$ is negative (because $\sqrt{5} > 1$), and x is a length, so we discard it. Thus $\boxed{AB = \dfrac{1+\sqrt{5}}{2} \approx 1.618}$ units. This number is called the **golden ratio**, and a rectangle of this shape is called a **golden rectangle**.

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Example 8. The diagonal of a rectangle is 17 cm long. The longer side of the rectangle is 1 cm less than twice the shorter side. How long are the sides of the rectangle?

Solution: Let x denote the length of the shorter side. Then the longer side is $2x - 1$. The diagonal splits the rectangle into two right triangles, and the diagonal is their hypotenuse. By the Pythagorean theorem,

$$\begin{aligned} x^2 + (2x-1)^2 &= 17^2 && \text{square the binomial} \\ x^2 + 4x^2 - 4x + 1 &= 289 \\ 5x^2 - 4x + 1 &= 289 && \text{combine like terms} \\ 5x^2 - 4x - 288 &= 0 && \text{subtract 289} \end{aligned}$$

We apply the quadratic formula with values $a = 5$, $b = -4$, and $c = -288$.

$$x_{1,2} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 5 \cdot (-288)}}{2 \cdot 5} = \frac{4 \pm \sqrt{16 + 5760}}{10} = \frac{4 \pm \sqrt{5776}}{10} = \frac{4 \pm 76}{10} = \begin{cases} \dfrac{80}{10} = 8 \\ \dfrac{-72}{10} = -\dfrac{36}{5} \end{cases}$$

A length can not be negative, so $x = 8$. The sides of the rectangle are $\boxed{8 \text{ cm and } 2 \cdot 8 - 1 = 15 \text{ cm}}$. We check: $8^2 + 15^2 = 64 + 225 = 289 = 17^2$.

Example 9. A bag contains 5 blue marbles and some red marbles. We draw two marbles at random, without replacement. The probability that both marbles are red is $\frac{3}{7}$. How many red marbles are in the bag?

Solution: Let r denote the number of red marbles. Then there are $r + 5$ marbles in the bag. By the AND rule, the probability that the first marble is red and the second marble is also red is $\frac{r}{r+5} \cdot \frac{r-1}{r+4}$, because after the first red marble is drawn, there are $r - 1$ red marbles among $r + 4$ marbles.

$$\begin{aligned} \frac{r(r-1)}{(r+5)(r+4)} &= \frac{3}{7} && \text{multiply by } 7(r+5)(r+4) \\ 7r(r-1) &= 3(r+5)(r+4) \\ 7r^2 - 7r &= 3r^2 + 27r + 60 && \text{multiply out both sides} \\ 4r^2 - 34r - 60 &= 0 && \text{subtract } 3r^2, 27r, \text{ and } 60 \\ 2r^2 - 17r - 30 &= 0 && \text{divide by 2} \end{aligned}$$

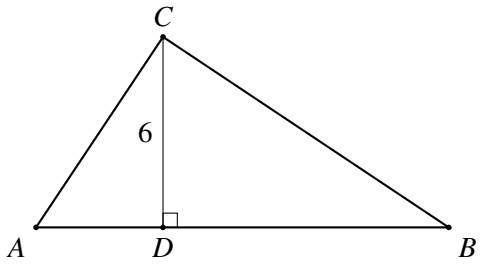
We apply the quadratic formula with values $a = 2$, $b = -17$, and $c = -30$.

$$r_{1,2} = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \cdot 2 \cdot (-30)}}{2 \cdot 2} = \frac{17 \pm \sqrt{289 + 240}}{4} = \frac{17 \pm \sqrt{529}}{4} = \frac{17 \pm 23}{4} = \begin{cases} \dfrac{40}{4} = 10 \\ \dfrac{-6}{4} = -\dfrac{3}{2} \end{cases}$$

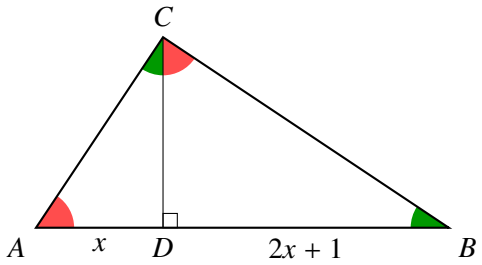
The number of marbles can not be negative, so there are $\boxed{10 \text{ red marbles}}$ in the bag. We check: with 10 red and 5 blue marbles, $\frac{10}{15} \cdot \frac{9}{14} = \frac{90}{210} = \frac{3}{7}$.

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Example 10. In the right triangle ABC, the right angle is at C. The altitude CD drawn to the hypotenuse is 6 units long. The point D splits the hypotenuse so that DB is 1 unit more than twice AD. Find the lengths of AD and DB.



Solution: The altitude CD splits the right triangle into two smaller right triangles, ADC and CDB. Let α denote the angle at A and β the angle at B. The triangle ADC has the angle α at A and a right angle at D, so its angle at C is β. In the same way, the triangle CDB has the angle β at B and the angle α at C. So the two small triangles have the same angles: they are similar.



Let x denote the length of AD. Then $DB = 2x + 1$. We write the ratio $\frac{\text{side opposite } \alpha}{\text{side opposite } \beta}$ in both small triangles. In triangle ADC, the side opposite α is $CD = 6$ and the side opposite β is $AD = x$. In triangle CDB, the side opposite α is $DB = 2x + 1$ and the side opposite β is $CD = 6$.

$$\begin{aligned} \frac{6}{x} &= \frac{2x+1}{6} && \text{multiply by } 6x \\ 36 &= x\,(2x+1) && \\ 36 &= 2x^2 + x && \text{distribute } x \\ 0 &= 2x^2 + x - 36 && \text{subtract } 36 \end{aligned}$$

We apply the quadratic formula with values $a = 2$, $b = 1$, and $c = -36$.

$$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-36)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1 + 288}}{4} = \frac{-1 \pm \sqrt{289}}{4} = \frac{-1 \pm 17}{4} = \begin{cases} \frac{16}{4} = 4 \\ \frac{-18}{4} = -\frac{9}{2} \end{cases}$$

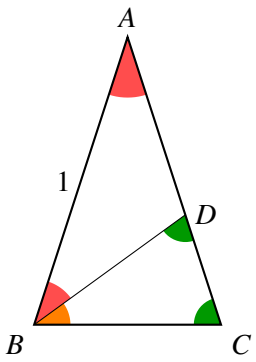
A length can not be negative, so $\boxed{AD = 4 \text{ and } DB = 2 \cdot 4 + 1 = 9}$. We check: $\frac{6}{4} = \frac{9}{6}$, both are $\frac{3}{2}$.

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Enrichment

The golden triangle. In the isosceles triangle ABC, $AB = AC = 1$ unit and the angle at A is 36°. The bisector of the angle at B meets the side AC at D. Find the exact value of the length of the base BC.



Solution: The base angles of the triangle ABC are $\dfrac{180^\circ - 36^\circ}{2} = 72^\circ$. The bisector BD cuts the 72° angle at B into two 36° angles. In the triangle BDC, the angles at B and C are 36° and 72°, so the angle BDC is 72°. The triangle ABD has two 36° angles, so $AD = BD$. The triangle BDC has two 72° angles, so $BD = BC$. Let x denote the length of BC. Then $BD = AD = x$ and $DC = 1 - x$.

The triangles ABC and BCD have the same angles $(36^\circ, 72^\circ, 72^\circ)$, so they are similar. We write the ratio $\dfrac{\text{leg}}{\text{base}}$ in both: $\dfrac{1}{x} = \dfrac{x}{1-x}$. We multiply both sides by $x\,(1-x)$ and get $1 - x = x^2$, so $x^2 + x - 1 = 0$. By the quadratic formula,

$$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$$

The negative solution is discarded, so $BC = \dfrac{-1 + \sqrt{5}}{2} \approx 0.618$. Its reciprocal is the golden ratio: $\dfrac{2}{\sqrt{5} - 1} = \dfrac{1 + \sqrt{5}}{2} \approx 1.618$. This triangle is called the **golden triangle**.

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Practice Problems

1. Use the discriminant to determine how many real solutions are there for each of the given equations. You do not have to find the solutions.

   a) $-3x^2 + 2x - 8 = 0$
   b) $5x^2 - 1 = 2x$
   c) $0.1x^2 + x + 1 = 0$
   d) $\frac{4}{9}x^2 - \frac{4}{3}x + 1 = 0$
   e) $x^2 - 6x = -7$
   f) $x^2 - 8x + 20 = 0$
   g) $-2x^2 - 5x + 10 = 0$

2. Use the quadratic formula to solve each of the following equations over the real numbers.

   a) $6x^2 - x + 9 = 0$
   b) $6x + 2x^2 + 1 = x + 4$
   c) $9x^2 - 12x = 1$
   d) $x + 40x^2 = 15$
   e) $-2x^2 + 5x - 4 = 0$
   f) $x^2 + x = 1$
   g) $\frac{16}{25}x^2 - \frac{16}{5}x + 4 = 0$

3. Find all positive numbers with the property that it is one greater than its own reciprocal.

4. Find all positive numbers with the property that it is two greater than its own reciprocal.

5. The diagonal of a rectangle is 13 cm long. The longer side of the rectangle is 2 cm more than twice the shorter side. How long are the sides of the rectangle?

6. A bag contains 6 blue marbles and some red marbles. We draw two marbles at random, without replacement. The probability that both marbles are red is $\frac{3}{8}$. How many red marbles are in the bag?

7. A bag contains 4 blue marbles and some red marbles. In a game, you draw two marbles at random, without replacement. If the two marbles have the same color, you win \$5. If they have different colors, you lose \$4. The game is fair: its expected value is \$0. How many red marbles are in the bag?

8. Two chords of a circle, AB and CD, intersect each other at point P. We know that $CP = 5$ cm and $PD = 8$ cm, and that PB is 2 cm less than three times AP. Find the lengths of AP and PB. (Hint: the triangles APC and DPB are similar.)

9. In the right triangle ABC, the right angle is at C. The altitude CD drawn to the hypotenuse is 12 units long. The point D splits the hypotenuse so that DB is 11 units less than three times AD. Find the lengths of AD and DB.

10. Solve the problem of Example 7 again, if $AD = 4$ units.

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Answers

1. a) no real solutions b) two real solutions c) two real solutions d) one real solution
   e) two real solutions f) no real solutions g) two real solutions

2. a) no real solutions b) $-3, \frac{1}{2}$ c) $\frac{2 \pm \sqrt{5}}{3}$ d) $-\frac{5}{8}, \frac{3}{5}$ e) no real solutions f) $\frac{-1 \pm \sqrt{5}}{2}$ g) $\frac{5}{2}$

3. $\frac{1+\sqrt{5}}{2}$ (the other solution is negative)

4. $1+\sqrt{2}$ (the other solution, $1-\sqrt{2}$, is negative)

5. 5 cm and 12 cm (from $5x^2 + 8x - 165 = 0$)

6. 10 red marbles (from $5r^2 - 41r - 90 = 0$; the other solution is $-\frac{9}{5}$)

7. 5 red marbles (the probability of the same color is $\frac{4}{9}$; from $5r^2 - 37r + 60 = 0$; the other solution, $\frac{12}{5}$, is not a whole number)

8. $AP = 4$ cm and $PB = 10$ cm (from $3x^2 - 2x - 40 = 0$)

9. $AD = 9$ and $DB = 16$ (from $3x^2 - 11x - 144 = 0$)

10. $AB = 2 + 2\sqrt{5} \approx 6.472$ units (from $x^2 - 4x - 16 = 0$)

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