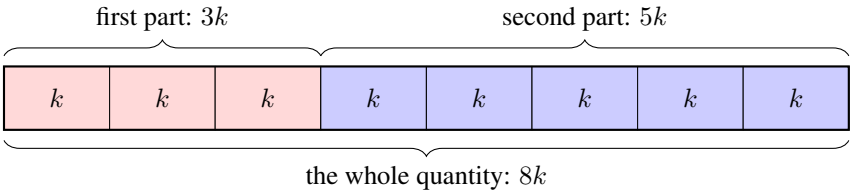
Two friends, Ana and Ben, start a small business together. Ana puts in \$3000 and Ben puts in \$5000. At the end of the year, the business makes a profit of \$1200, and they agree to share it fairly: in the same ratio as they invested, $3000 : 5000 = 3 : 5$. How much does each of them get?

The trick is to think of the profit as cut into $3 + 5 = 8$ equal pieces. Ana gets 3 of the pieces and Ben gets 5 of them. If we call the size of one piece k, then Ana gets $3k$ dollars and Ben gets $5k$ dollars.



Splitting a quantity in a given ratio: To split a quantity Q into two parts in the ratio $a : b$, we write the two parts as ak and bk. Together they make up the whole quantity, so

$$ak + bk = Q \qquad \text{or} \qquad (a + b)\, k = Q$$

We solve this equation for k, and then compute the two parts, ak and bk. The same works for three or more parts.

Example 1. Ana and Ben share a profit of \$1200 in the ratio $3 : 5$. How much does each of them get?

Solution: We write Ana's share as $3k$ and Ben's share as $5k$ dollars. Together they make up the whole profit:

$$3k + 5k = 1200 \quad \Longrightarrow \quad 8k = 1200 \quad \Longrightarrow \quad k = 150$$

So Ana gets $3 \cdot 150 = \boxed{\$450}$ and Ben gets $5 \cdot 150 = \boxed{\$750}$. Check: $450 + 750 = 1200$, and $\dfrac{450}{750} = \dfrac{3}{5}$.

Example 2. A lemonade recipe mixes lemonade concentrate and water in the ratio $2 : 7$. How much concentrate and how much water do we need for 45 cups of lemonade?

Solution: We need $2k$ cups of concentrate and $7k$ cups of water.

$$2k + 7k = 45 \quad \Longrightarrow \quad 9k = 45 \quad \Longrightarrow \quad k = 5$$

So we need $2 \cdot 5 = \boxed{\text{10 cups of concentrate}}$ and $7 \cdot 5 = \boxed{\text{35 cups of water}}$.

Example 3. A 54-inch board is cut into three pieces whose lengths are in the ratio $2 : 3 : 4$. How long is each piece?

Solution: The three pieces are $2k$, $3k$, and $4k$ inches long.

$$2k + 3k + 4k = 54 \quad \Longrightarrow \quad 9k = 54 \quad \Longrightarrow \quad k = 6$$

So the pieces are $2 \cdot 6 = 12$, $3 \cdot 6 = 18$, and $4 \cdot 6 = 24$ inches long: $\boxed{\text{12 in, 18 in, and 24 in}}$.

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Example 4. Split 70 into two parts in the ratio a) $1.5 : 2$ b) $\frac{2}{3} : \frac{1}{2}$

Solution: A ratio does not change if we multiply both of its numbers by the same (nonzero) number. So we first turn the ratio into a ratio of whole numbers.

a) We multiply both numbers by 2: $1.5 : 2 = 3 : 4$. So the parts are $3k$ and $4k$, and $7k = 70$, so $k = 10$. The two parts are $\boxed{30 \text{ and } 40}$.

b) We multiply both numbers by 6, the common denominator: $\frac{2}{3} : \frac{1}{2} = 4 : 3$. So the parts are $4k$ and $3k$, and $7k = 70$, so $k = 10$. The two parts are $\boxed{40 \text{ and } 30}$.

Example 5. Two friends share some money in the ratio $4 : 7$. The larger share is \$350. How much is the smaller share, and how much money did they share altogether?

Solution: This time we know one of the parts, not the whole. The shares are $4k$ and $7k$, and the larger one is $7k$.

$$7k = 350 \qquad \Longrightarrow \qquad k = 50$$

So the smaller share is $4 \cdot 50 = \boxed{\$200}$, and altogether they shared $200 + 350 = \boxed{\$550}$.

Example 6. A jar contains red and blue marbles in the ratio $3 : 5$. After we add 6 red marbles to the jar, there are as many red marbles as blue ones. How many red and how many blue marbles were in the jar originally?

Solution: Originally there were $3k$ red and $5k$ blue marbles. After adding 6 red marbles, there are $3k + 6$ red marbles, and this equals the number of blue marbles.

$$\begin{aligned} 3k + 6 &= 5k && \text{We subtract } 3k \text{ from both sides.} \\ 6 &= 2k && \text{We divide both sides by 2.} \\ 3 &= k \end{aligned}$$

So originally there were $3 \cdot 3 = \boxed{9 \text{ red}}$ and $5 \cdot 3 = \boxed{15 \text{ blue}}$ marbles. Check: after adding 6 red marbles, there are 15 red and 15 blue marbles.

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When we roll a die, the probability of rolling a six is $\frac{1}{6}$. Sports fans and gamblers often say the same thing differently: the **odds** of rolling a six are 1 to 5. This means that for every 1 outcome that is good for us, there are 5 outcomes that are not.

Definition: Suppose that the outcomes of an experiment are equally likely, and the event E contains f of them (the favorable outcomes) while u of them are not in E (the unfavorable outcomes). Then

the odds in favor of E are $f : u$ and the odds against E are $u : f$.

Odds are ratios, so we can simplify them like fractions.

Example 7. We roll a die. What are the odds in favor of rolling a number greater than 4? What are the odds against it?

Solution: There are 2 favorable outcomes (5 and 6) and 4 unfavorable ones (1, 2, 3, and 4). So the odds in favor are $2 : 4 = \boxed{1 : 2}$, and the odds against are $\boxed{2 : 1}$. The probability of rolling a number greater than 4 is $\frac{2}{6} = \frac{1}{3}$.

Example 8. We pull a card from a well-shuffled deck of 52 cards. What are the odds in favor of pulling a heart?

Solution: There are 13 hearts and $52 - 13 = 39$ cards that are not hearts. So the odds in favor of a heart are $13 : 39 = \boxed{1 : 3}$. The probability of a heart is $\frac{13}{52} = \frac{1}{4}$.

Careful! The odds $1 : 3$ do NOT mean a probability of $\frac{1}{3}$. The odds compare the favorable outcomes to the **unfavorable** ones, while the probability compares the favorable outcomes to **all** of them. Odds of $1 : 3$ mean 1 favorable outcome out of $1 + 3 = 4$, so the probability is $\frac{1}{4}$.

This gives us the connection between odds and probabilities. If the odds in favor of E are $a : b$, then the outcomes are split into favorable and unfavorable ones in the ratio $a : b$, just like a quantity split in a ratio. There are ak favorable and bk unfavorable outcomes, $(a + b)\, k$ altogether, so

$$P(E) = \frac{ak}{(a+b)\,k} = \frac{a}{a+b}$$

From odds to probability: If the odds in favor of E are $a : b$, then $P(E) = \frac{a}{a+b}$.

From probability to odds: If $P(E) = \frac{m}{n}$, then the odds in favor of E are $m : (n - m)$.

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Example 9. A bag contains red and green marbles. We pull one marble at random, and the odds in favor of a red marble are $2 : 5$.

a) What is the probability that the marble is red? b) If there are 35 marbles in the bag, how many of them are red?

Solution: a) $P(\text{red}) = \dfrac{2}{2+5} = \boxed{\dfrac{2}{7}}$

b) The marbles are split into red and green ones in the ratio $2 : 5$. There are $2k$ red and $5k$ green marbles, so $7k = 35$, and $k = 5$. There are $2 \cdot 5 = \boxed{\text{10 red marbles}}$.

Example 10. The probability of an event is 0.35. What are the odds in favor of the event, and the odds against it?

Solution: $0.35 = \dfrac{35}{100} = \dfrac{7}{20}$. Out of every 20 outcomes, 7 are favorable and $20 - 7 = 13$ are not. So the odds in favor are $\boxed{7 : 13}$, and the odds against are $\boxed{13 : 7}$.

Example 11. We roll two dice. What are the odds in favor of rolling a sum of 7?

Solution: There are $6 \cdot 6 = 36$ equally likely outcomes. The sum is 7 in 6 of them: $1 + 6$, $2 + 5$, $3 + 4$, $4 + 3$, $5 + 2$, and $6 + 1$. So there are 6 favorable and 30 unfavorable outcomes, and the odds in favor are $6 : 30 = \boxed{1 : 5}$.



Enrichment

Fair bets. We pay \$1 to play a game: we roll a die, and if it shows a six, we get our dollar back plus a prize of X dollars. If it does not show a six, we lose our dollar. How large should the prize be to make the game fair, so that the expected value of our winnings is zero?

We win X dollars with probability $\dfrac{1}{6}$, and we lose 1 dollar with probability $\dfrac{5}{6}$.

$$\frac{1}{6} \cdot X - \frac{5}{6} \cdot 1 = 0 \qquad \Longrightarrow \qquad \frac{1}{6} \cdot X = \frac{5}{6} \qquad \Longrightarrow \qquad X = 5$$

So a fair game pays \$5 for each \$1 we bet. This is exactly the odds against winning: the odds against a six are $5 : 1$. In general, a bet is fair if it pays according to the odds against winning. Casinos always pay a little less than that; this small difference is how they make their money.

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Practice Problems

1. Split \$840 between two people in the ratio $3 : 4$.
2. A 25-pound mixture contains almonds and cashews in the ratio $2 : 3$. How many pounds of each nut are in the mixture?
3. Split 96 into three parts in the ratio $1 : 3 : 4$.
4. The angles of a triangle are in the ratio $2 : 3 : 4$. Find the angles.
5. Split 60 into two parts in the ratio
   a) $2.5 : 3.5$ b) $\frac{1}{2} : \frac{1}{3}$
6. Two friends share a prize in the ratio $5 : 8$. The smaller share is \$1250. Find the larger share and the whole prize.
7. A bag contains red and green marbles in the ratio $4 : 7$. After we remove 6 green marbles, there are as many red marbles as green ones. How many red and how many green marbles were in the bag originally?
8. We roll a die. What are the odds in favor of rolling a number less than 3? What are the odds against it?
9. We pull a card from a well-shuffled deck of 52 cards. What are the odds in favor of pulling a face card (a jack, queen, or king)? What is the probability of pulling a face card?
10. The odds in favor of an event E are $4 : 9$. Find the probability of E, and the probability that E does not happen.
11. The probability of an event is 0.6. What are the odds in favor of the event, and the odds against it?
12. A bag contains 40 marbles, some blue and some white. We pull a marble at random, and the odds in favor of a blue marble are $3 : 5$. How many blue marbles are in the bag?
13. We toss two coins. What are the odds in favor of getting at least one head?
14. (Enrichment) We bet \$1 that the sum of two dice will be 7. How much should a fair game pay us if we win?

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Answers

1. $3k + 4k = 840$, $k = 120$: \$360 and \$480
2. $2k + 3k = 25$, $k = 5$: 10 pounds of almonds and 15 pounds of cashews
3. $k + 3k + 4k = 96$, $k = 12$: 12, 36, and 48
4. The angles of a triangle add up to 180°: $2k + 3k + 4k = 180$, $k = 20$: 40°, 60°, and 80°
5. a) $2.5 : 3.5 = 5 : 7$, $12k = 60$, $k = 5$: 25 and 35 b) $\frac{1}{2} : \frac{1}{3} = 3 : 2$, $5k = 60$, $k = 12$: 36 and 24
6. $5k = 1250$, $k = 250$: the larger share is \$2000, and the whole prize is \$3250
7. $4k = 7k - 6$. We subtract $4k$ from both sides: $0 = 3k - 6$, we add 6 to both sides: $6 = 3k$, so $k = 2$: 8 red and 14 green marbles
8. 2 favorable (1 and 2), 4 unfavorable: odds in favor $2 : 4 = 1 : 2$, odds against $2 : 1$
9. 12 face cards, 40 other cards: odds in favor $12 : 40 = 3 : 10$, $P = \frac{12}{52} = \frac{3}{13}$
10. $P(E) = \frac{4}{4+9} = \frac{4}{13}$, $P(\text{not } E) = \frac{9}{13}$
11. $0.6 = \frac{3}{5}$: odds in favor $3 : 2$, odds against $2 : 3$
12. $3k + 5k = 40$, $k = 5$: 15 blue marbles
13. HH, HT, TH, TT: 3 favorable, 1 unfavorable, odds in favor $3 : 1$
14. $P(7) = \frac{6}{36} = \frac{1}{6}$, so the odds against are $5 : 1$, and a fair game pays \$5 (plus our dollar back).

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