

Today we studied two-step equations and how a longer equation, once we distribute and combine like terms, often turns out to be a two-step equation in disguise. This handout collects both ideas in one place, together with several application problems.

Part 1 – Two-Step Equations

Suppose we decide to hide a small object, say a coin. We put the coin on the table, then place an envelope over it, and then, just to be sure, we place a hat on top of the envelope. Let us find the coin! To do that, what do we need to remove, and in what order? We would first remove the hat and then the envelope, right?

This is the basis of solving two-step equations. To isolate the unknown, we will perform the inverse operations, in the reverse order. What happened last can be undone first.

Example 1. Solve each of the given equations. Make sure to check your solutions.

$$\text{a) } 10 = 3x - 11 \quad \text{b) } 3x + 8 = -7 \quad \text{c) } \frac{t-7}{2} = -8 \quad \text{d) } \frac{x}{-3} + 4 = 15$$

Solution: a) The equation $10 = 3x - 11$ looks unusual in the sense that two-step equations often contain the unknown on the left-hand side. We are always allowed to swap two sides of an equation. If $A = B$, then clearly, also $B = A$. We will do this first. This is an optional step that is always available.

$$\begin{aligned} 10 &= 3x - 11 && \text{we swap the two sides} \\ 3x - 11 &= 10 \end{aligned}$$

We now look at the side that contains x and ask: *What happened to the unknown?* The answer is: *Multiplication by 3 and then subtraction of 11.* We need to apply the inverse operations, in reverse order. In this case, this means that we will add 11 to both sides and then divide both sides by 3.

$$\begin{aligned} 3x - 11 &= 10 && \text{add 11} \\ 3x &= 21 && \text{divide by 3} \\ x &= 7 \end{aligned}$$

So the only solution of this equation is 7. We check: if $x = 7$, then

$$\text{LHS} = 3x - 11 = 3 \cdot 7 - 11 = 21 - 11 = 10 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = 7}$ is correct.

b) As we look at the equation $3x + 8 = -7$, we ask: *What happened to the unknown?* The answer is: *Multiplication by 3 and then addition of 8.* We need to apply the inverse operations, in a reverse order. In this case, this means that we will subtract 8 from both sides and then divide both sides by 3.

$$\begin{aligned} 3x + 8 &= -7 && \text{subtract 8} \\ 3x &= -15 && \text{divide by 3} \\ x &= -5 \end{aligned}$$

So the only solution of this equation is -5 . We check: if $x = -5$, then

$$\text{LHS} = 3x + 8 = 3(-5) + 8 = -15 + 8 = -7 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = -5}$ is correct.

- c) What happened to the unknown? On the left-hand side, there was a subtraction of 7 and then a division by 2. To reverse that, we will multiply both sides by 2 and then add 7 to both sides.

$$\begin{array}{rcl} \frac{t-7}{2} & = & -8 \quad \text{multiply by 2} \\ t-7 & = & -16 \quad \text{add 7} \\ t & = & -9 \end{array}$$

So the only solution of this equation is -9 . We check: if $t = -9$, then

$$\text{LHS} = \frac{t-7}{2} = \frac{-9-7}{2} = \frac{-16}{2} = -8 = \text{RHS} \checkmark$$

So our solution, $t = -9$ is correct.

- d) What happened to the unknown? On the left-hand side, there was a division by -3 and then an addition of 4. To reverse that, we will subtract 4 from both sides and then multiply both sides by -3 .

$$\begin{array}{rcl} \frac{x}{-3} + 4 & = & 15 \quad \text{subtract 4} \\ \frac{x}{-3} & = & 11 \quad \text{multiply by } -3 \\ x & = & -33 \end{array}$$

So the only solution of this equation is -33 . We check: if $x = -33$, then

$$\text{LHS} = \frac{x}{-3} + 4 = \frac{-33}{-3} + 4 = 11 + 4 = 15 = \text{RHS} \checkmark$$

So our solution, $x = -33$ is correct.

Example 2. The sum of three times a number and seven is -5 . Find this number.

Solution: Let us denote our mystery number by x . The equation will be just the first sentence, translated to algebra. The sum of three times the number and seven is $3x + 7$. So our equation is $3x + 7 = -5$. We will know the number if we solve this equation.

$$\begin{array}{rcl} 3x + 7 & = & -5 \quad \text{subtract 7} \\ 3x & = & -12 \quad \text{divide by 3} \\ x & = & -4 \end{array}$$

Good news! We do not need to check if -4 is indeed the solution of the equation. What if we correctly solved the *wrong* equation? Recall that *we* came up with the equation, it was not given. Instead of checking the number against the equation, we should check if our solution satisfies the conditions stated in the problem. Is it true the sum of three times -4 and seven is -5 ? Indeed, $3(-4) + 7 = -12 + 7 = -5$. Thus our solution, -4 , is correct.

Example 3. The age of my sister is five less than twice my age. If she is 35, how old am I?

Solution: Let us denote my age by x . Twice my age is $2x$, and five less than that is $2x - 5$. This must be my sister's age,

35. So our equation is $2x - 5 = 35$.

$$\begin{array}{rcl} 2x - 5 & = & 35 \quad \text{add 5} \\ 2x & = & 40 \quad \text{divide by 2} \\ x & = & 20 \end{array}$$

We check: twice my age is $2(20) = 40$, and five less than that is $40 - 5 = 35$, which is indeed my sister's age. Thus I am 20 years old.

Part 2 – Multi-Step Equations

Many equations look more complicated than the ones we just solved because they contain parentheses, or because the unknown appears more than once on the same side. The good news is that once we apply the distributive law and combine like terms, these equations reduce to the two-step equations we already know how to solve.

Example 4. Solve the equation $5(3x - 8) - 2(8x - 1) = -33$. Make sure to check your solution.

Solution: We need to simplify the left-hand side. After that, the equation will be a simple two-step equation.

$$\begin{array}{rcl} 5(3x - 8) - 2(8x - 1) & = & -33 \quad \text{distribute 5 and } -2 \\ 15x - 40 - 16x + 2 & = & -33 \quad \text{combine like terms} \\ -x - 38 & = & -33 \end{array}$$

This is now a two-step equation we can easily solve for x .

$$\begin{array}{rcl} -x - 38 & = & -33 \quad \text{add 38} \\ -x & = & 5 \quad \text{divide (or multiply) by } -1 \\ x & = & -5 \end{array}$$

We check: if $x = -5$, then

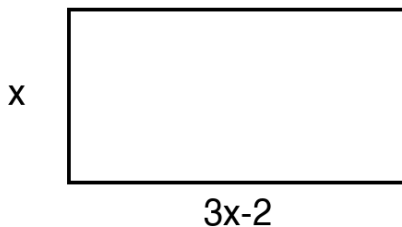
$$\begin{aligned} \text{LHS} &= 5[3(-5) - 8] - 2[8(-5) - 1] \\ &= 5(-23) - 2(-41) = -115 + 82 = -33 = \text{RHS } \checkmark \end{aligned}$$

Thus our solution, $x = -5$, is correct.

As before, we can use this idea to solve application problems. The steps are the same ones we already use: label the unknown, label all other quantities in terms of it, write an equation using the one piece of information not yet used, and then solve.

Example 5. One side of a rectangle is two feet shorter than three times another side. Find the sides of the rectangle if its perimeter is 20 feet.

Solution: Let us denote the shorter side by x . Then the longer side can be expressed as $3x - 2$.



We will write and solve an equation expressing the perimeter of the rectangle. The perimeter of a rectangle with

sides a and b is $P = 2a + 2b$.

$$2 \underbrace{x}_{\text{shorter side}} + 2 \underbrace{(3x - 2)}_{\text{longer side}} = 20$$

We simplify the left-hand side by applying the distributive law and then combining like terms.

$$\begin{aligned} 2x + 2(3x - 2) &= 20 && \text{distribute 2} \\ 2x + 6x - 4 &= 20 && \text{combine like terms} \\ 8x - 4 &= 20 \end{aligned}$$

This is now a two-step equation we can easily solve for x .

$$\begin{aligned} 8x - 4 &= 20 && \text{add 4} \\ 8x &= 24 && \text{divide by 8} \\ x &= 3 \end{aligned}$$

The shorter side was labeled x , so it is 3 feet. The longer side was labeled $3x - 2$, so it is $3(3) - 2 = 7$ feet. We check: is the longer side indeed two feet shorter than three times the shorter side? Indeed, $3(3) - 2 = 7 \checkmark$ Is the perimeter 20 feet? Indeed, $2(3) + 2(7) = 6 + 14 = 20 \checkmark$ So the sides are 3 feet and 7 feet long.

Example 6. The tickets for a field trip were purchased for both children and adults. Children tickets cost \$12, adult tickets cost \$19. The number of children tickets purchased was three more than four times the number of adult tickets purchased. How many of each were purchased if all of the tickets cost a total of \$304?

Solution: Let us denote the number of adult tickets by x . Then the number of children tickets is $4x + 3$. We organize our data using a table.

	adult tickets	children tickets
number of tickets	x	$4x + 3$
value in dollars	$19x$	$12(4x + 3)$

The equation will express the total value of the tickets. We solve the equation.

$$\begin{aligned} 19x + 12(4x + 3) &= 304 && \text{distribute 12} \\ 19x + 48x + 36 &= 304 && \text{combine like terms} \\ 67x + 36 &= 304 && \text{subtract 36} \\ 67x &= 268 && \text{divide by 67} \\ x &= 4 \end{aligned}$$

Since we labeled the number of adult tickets by x , there are 4 adult tickets. The number of children tickets was labeled $4x + 3$, thus there are $4(4) + 3 = 19$ children tickets. Our answer is: 4 adult tickets and 19 children tickets. We check: $19(4) + 12(19) = 76 + 228 = 304$ dollars. Thus our answer is correct.

Example 7. We purchased sodas and sandwiches. Sodas cost \$2, sandwiches cost \$5. The number of sandwiches purchased is 3 less than twice the number of sodas purchased. How much of each did we buy if we paid a total of \$105?

Solution: Let us denote the number of sodas by x . Then the number of sandwiches is $2x - 3$. We organize our data using a table.

	sodas	sandwiches
number purchased	x	$2x - 3$
value in dollars	$2x$	$5(2x - 3)$

The equation will express the total amount paid. We solve the equation.

$$2x + 5(2x - 3) = 105 \quad \text{distribute 5}$$

$$\begin{array}{ll}
 2x + 10x - 15 = 105 & \text{combine like terms} \\
 12x - 15 = 105 & \text{add 15} \\
 12x = 120 & \text{divide by 12} \\
 x = 10 &
 \end{array}$$

Since we labeled the number of sodas by x , we bought 10 sodas. The number of sandwiches was labeled $2x - 3$, thus we bought $2(10) - 3 = 17$ sandwiches. Our answer is: 10 sodas and 17 sandwiches. We check: $2(10) + 5(17) = 20 + 85 = 105$ dollars. Thus our answer is correct.



Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 5 = 17$
2. $\frac{a - 10}{5} = -3$
3. $3(x + 7) = 36$
4. $-4x + 6 = -18$
5. $2(3x - 1) - 4(x - 2) = 14$
6. $5(2x - 3) - 3(3x - 1) = -2$

Solve each of the following application problems.

1. The age of my brother is four less than three times my age. If he is 23, how old am I?
2. One side of a rectangle is three feet longer than twice another side. Find the sides of the rectangle if its perimeter is 54 feet.



Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 3 = -11$
2. $-2x - 3 = 7$
3. $5x - 3 = 17$
4. $3b + 13 = -5$
5. $3(4x - 1) - 5(2x + 3) = -20$
6. $4(5x - 3(2x - 1) + 2x) = 52$
7. $3(x - 4) - 2(x + 1) = 0$
8. $2(3x + 1) - (x - 5) = 32$

Solve each of the following application problems.

1. Susan was asked about her age. She answered as follows: *My big brother's age is six less than three times my age.* How old is Susan if her big brother is 21 years old?
2. Amy's age is three less than five times her son's age. How old are they if the sum of their ages is 33?
3. Stan has five more textbooks than fiction books. How many of each does he have if all together he has 93 books?
4. A school purchases tickets to a show. A child ticket costs \$8 and an adult ticket costs \$14. The number of child tickets was five greater than three times the number of adult tickets. The school paid a total of \$610 for tickets. How many of each kind of ticket were purchased?



Answers

Sample Problems

1. 11 2. -5 3. 5 4. 6 5. 4 6. 10
7. I am 9 years old. 8. The sides are 8 feet and 19 feet.

Practice Problems

1. -4 2. -5 3. 4 4. -6 5. -1 6. 10 7. 14 8. 5
9. Susan is 9 years old. 10. Amy is 27 and her son is 6. 11. Stan has 49 textbooks and 44 fiction books.
12. 15 adult tickets and 50 child tickets.

Sample Problems



Solutions

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 5 = 17$

Solution:

$$\begin{aligned} 2x - 5 &= 17 && \text{add 5} \\ 2x &= 22 && \text{divide by 2} \\ x &= 11 \end{aligned}$$

We check: if $x = 11$, then

$$\text{LHS} = 2(11) - 5 = 22 - 5 = 17 = \text{RHS} \checkmark$$

Thus our solution, $x = 11$, is correct.

2. $\frac{a - 10}{5} = -3$

Solution:

$$\begin{aligned} \frac{a - 10}{5} &= -3 && \text{multiply by 5} \\ a - 10 &= -15 && \text{add 10} \\ a &= -5 \end{aligned}$$

We check: if $a = -5$, then

$$\text{LHS} = \frac{-5 - 10}{5} = \frac{-15}{5} = -3 = \text{RHS} \checkmark$$

Thus our solution, $a = -5$, is correct.

3. $3(x + 7) = 36$

Solution: We apply all operations to both sides.

$$\begin{array}{rcl} 3(x + 7) & = & 36 & \text{divide by 3} \\ x + 7 & = & 12 & \text{subtract 7} \\ x & = & 5 \end{array}$$

We check: if $x = 5$, then

$$\text{LHS} = 3(5 + 7) = 3 \cdot 12 = 36 = \text{RHS} \checkmark$$

Thus our solution, $x = 5$, is correct.

4. $-4x + 6 = -18$

Solution:

$$\begin{array}{rcl} -4x + 6 & = & -18 & \text{subtract 6} \\ -4x & = & -24 & \text{divide by } -4 \\ x & = & 6 \end{array}$$

We check: if $x = 6$, then

$$\text{RHS} = -4(6) + 6 = -24 + 6 = -18 = \text{LHS} \checkmark$$

Thus our solution, $x = 6$, is correct.

5. $2(3x - 1) - 4(x - 2) = 14$

Solution: We distribute and combine like terms; the equation then becomes a two-step equation.

$$\begin{array}{rcl} 2(3x - 1) - 4(x - 2) & = & 14 & \text{distribute 2 and } -4 \\ 6x - 2 - 4x + 8 & = & 14 & \text{combine like terms} \\ 2x + 6 & = & 14 & \text{subtract 6} \\ 2x & = & 8 & \text{divide by 2} \\ x & = & 4 \end{array}$$

We check: if $x = 4$, then

$$\text{LHS} = 2(3 \cdot 4 - 1) - 4(4 - 2) = 2(11) - 4(2) = 22 - 8 = 14 = \text{RHS} \checkmark$$

Thus our solution, $x = 4$, is correct.

6. $5(2x - 3) - 3(3x - 1) = -2$

Solution: We distribute and combine like terms; the equation then becomes a two-step equation.

$$\begin{array}{rcl} 5(2x - 3) - 3(3x - 1) & = & -2 & \text{distribute 5 and } -3 \\ 10x - 15 - 9x + 3 & = & -2 & \text{combine like terms} \\ x - 12 & = & -2 & \text{add 12} \\ x & = & 10 \end{array}$$

We check: if $x = 10$, then

$$\text{LHS} = 5(2 \cdot 10 - 3) - 3(3 \cdot 10 - 1) = 5(17) - 3(29) = 85 - 87 = -2 = \text{RHS} \checkmark$$

Thus our solution, $x = 10$, is correct.

7. The age of my brother is four less than three times my age. If he is 23, how old am I?

Solution: Let us denote my age by x . Three times my age is $3x$, and four less than that is $3x - 4$. This must be my brother's age, 23. So our equation is $3x - 4 = 23$.

$$\begin{array}{rcll} 3x - 4 & = & 23 & \text{add 4} \\ 3x & = & 27 & \text{divide by 3} \\ x & = & 9 & \end{array}$$

We check: three times my age is $3(9) = 27$, and four less than that is $27 - 4 = 23$, which is indeed my brother's age. Thus $\boxed{\text{I am 9 years old}}$.

8. One side of a rectangle is three feet longer than twice another side. Find the sides of the rectangle if its perimeter is 54 feet.

Solution: Let us denote the shorter side by x . Then the longer side can be expressed as $2x + 3$. We write and solve an equation expressing the perimeter of the rectangle.

$$\begin{array}{rcll} 2x + 2(2x + 3) & = & 54 & \text{distribute 2} \\ 2x + 4x + 6 & = & 54 & \text{combine like terms} \\ 6x + 6 & = & 54 & \text{subtract 6} \\ 6x & = & 48 & \text{divide by 6} \\ x & = & 8 & \end{array}$$

The shorter side was labeled x , so it is 8 feet. The longer side was labeled $2x + 3$, so it is $2(8) + 3 = 19$ feet. We check: is the longer side indeed three feet longer than twice the shorter side? Indeed, $2(8) + 3 = 19 \checkmark$ Is the perimeter 54 feet? Indeed, $2(8) + 2(19) = 16 + 38 = 54 \checkmark$ So the sides are $\boxed{8 \text{ feet and } 19 \text{ feet}}$ long.

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