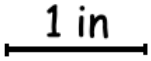
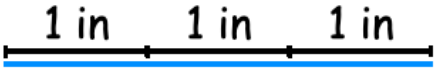
Perimeter is a measure of length. The length of an object, measured in inches, is the number of 1-inch line segments we need to cover our object.



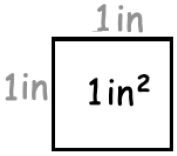
unit inch



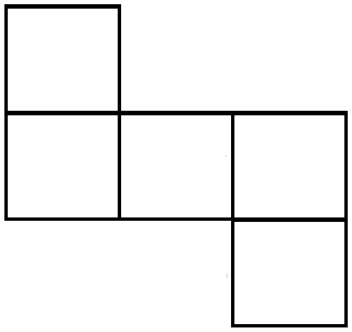
This blue line is 3 inches long.

Lengths are measured in meters (m), inches (in), feet (ft), miles (mi), and so on.

Area is a measure of surface. The area of an object, measured in square-inches, is the number of 1-inch by 1-inch squares we need to cover our object.



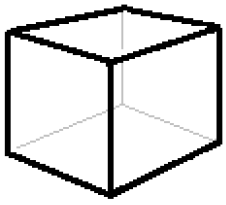
1 square-inch



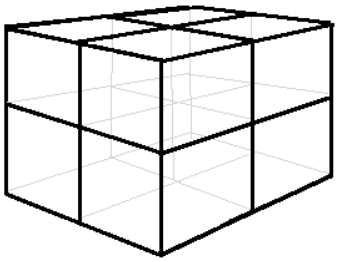
This area is 5 square-inches.

Area is measured in square-meters (m^2), square-inches (in^2), square-feet (ft^2), and so on.

Volume is a measure of space. The volume of an object, measured in cubic inches, is the number of 1-inch by 1-inch by 1-inch cubes we need to build a replica of our object. Another way to imagine this is to make an ice-sculpture replica of our object, melt it, and then freeze it into unit cubes and count them.



1 cubic inch



The volume of this object is 8 cubic inches.

Volumes are measured in cubic meters (m^3), cubic inches (in^3), cubic feet (ft^3), and so on.

Another way of thinking of all this is that length is one-dimensional (part of a line), area is two-dimensional (part of a plane), and volume is three-dimensional (part of space). The space we live in is three-dimensional.

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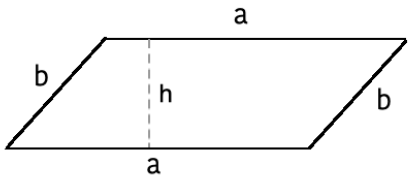
An Easy Way to Remember Volume Formulas

Let us call a shape a **solid** if it starts in some shape and ends in the same shape, and a **pointy** object if it starts in a shape and ends in a point. In the plane, for example, a parallelogram is a solid and a triangle is a pointy object. In space, a cylinder is a solid and a cone is a pointy object.

2-dimensional solid

Area is base times height

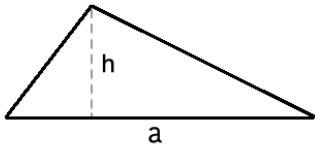
$$A = ah$$



2-dimensional pointy

Area is base times height over 2

$$A = \frac{ah}{2}$$

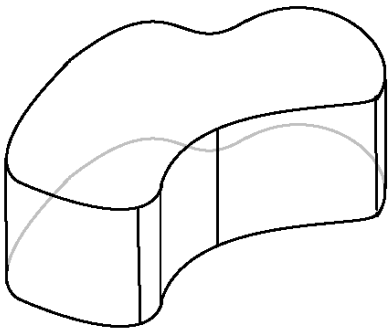


3-dimensional solid

Volume is base area times height

$$V = Bh$$

where B is the area of the base

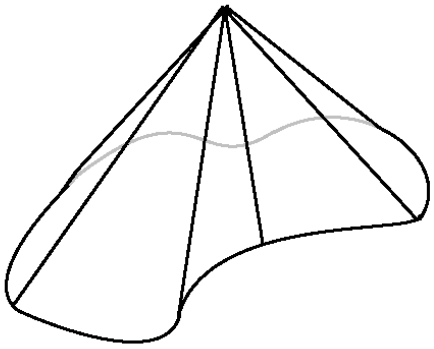


3-dimensional pointy

Volume is base area times height over 3

$$V = \frac{Bh}{3}$$

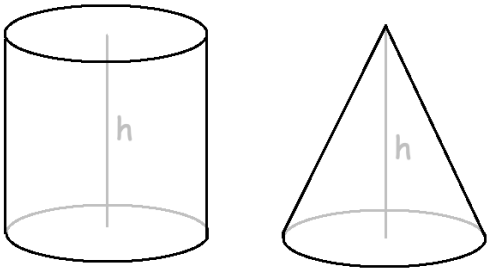
where B is the area of the base



Maybe we cannot visualize objects in four dimensions, but their "volume" formula is probably $\frac{Vh}{4}$: base-volume times the height, over four. Either way, the point here is that using this general rule is easier than memorizing many volume formulas. Always remember that the base area B must be computed first, and that the height h is measured at a right angle to the base.

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Example 1. The solid in the picture is called a cylinder. The pointy object is called a cone. Find the volume of each if the base is a circle with area $30\,\text{in}^2$ and the height is $12\,\text{in}$.



Solution. The area of the base is given, so we can apply our new volume formulas right away.

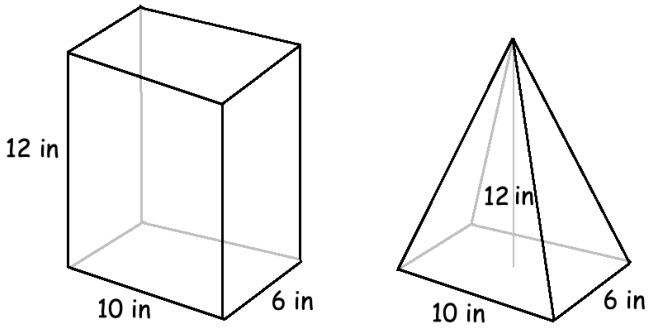
For the cylinder,

$$V = Bh = 30\,\text{in}^2 \cdot 12\,\text{in} = 360\,\text{in}^3.$$

For the cone,

$$V = \frac{Bh}{3} = \frac{30\,\text{in}^2 \cdot 12\,\text{in}}{3} = 120\,\text{in}^3.$$

Example 2. The solid in the picture is a rectangular prism. The pointy object is a rectangular pyramid. Find the volume of each.



Solution. In both cases, the base is a rectangle with sides $6\,\text{in}$ and $10\,\text{in}$, so its area is $B = 6\,\text{in} \cdot 10\,\text{in} = 60\,\text{in}^2$. The height is, in both cases, $12\,\text{in}$.

For the rectangular prism,

$$V = Bh = 60\,\text{in}^2 \cdot 12\,\text{in} = 720\,\text{in}^3.$$

For the pyramid,

$$V = \frac{Bh}{3} = \frac{60\,\text{in}^2 \cdot 12\,\text{in}}{3} = 240\,\text{in}^3.$$

Example 3. Find the volume of a cylinder and of a cone, if the base is a circle of radius $3\,\text{in}$ and the height is $10\,\text{in}$. Give the exact answer and an approximation rounded to two decimal places.

Solution. The area of the base is $B = \pi r^2 = \pi \cdot 3^2 = 9\pi\,\text{in}^2$. For the cylinder,

$$V = Bh = 9\pi\,\text{in}^2 \cdot 10\,\text{in} = 90\pi\,\text{in}^3 \approx 282.74\,\text{in}^3.$$

For the cone,

$$V = \frac{Bh}{3} = \frac{9\pi\,\text{in}^2 \cdot 10\,\text{in}}{3} = 30\pi\,\text{in}^3 \approx 94.25\,\text{in}^3.$$

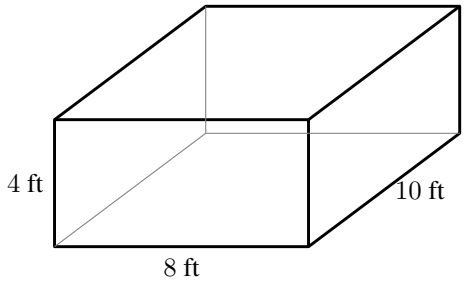
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Example 4. A cylindrical water tank has a diameter of 6 feet and a height of 5 feet. How many cubic feet of water does it hold when it is full?

Solution. The diameter is 6 feet, so the radius is 3 feet. The area of the base is $B = \pi \cdot 3^2 = 9\pi \text{ ft}^2$ and

$$V = Bh = 9\pi \text{ ft}^2 \cdot 5 \text{ ft} = 45\pi \text{ ft}^3 \approx 141.37 \text{ ft}^3.$$

Example 5. A sandbox is 8 feet long and 10 feet wide, and the sand in it is 4 feet deep. How many cubic feet of sand are in the sandbox? How many 2-cubic-foot bags of sand were needed to fill it?

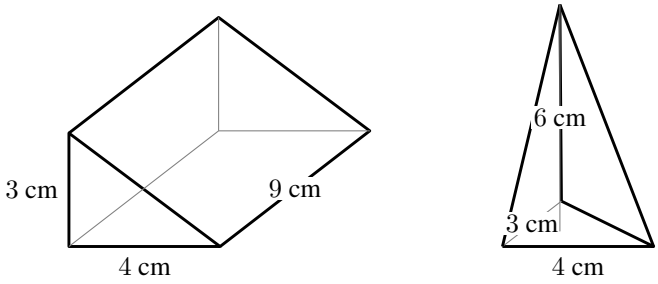


Solution. The sandbox is a rectangular prism. The base is a rectangle 8 feet by 10 feet, so $B = 80 \text{ ft}^2$, and the height is the depth of the sand.

$$V = Bh = 80 \text{ ft}^2 \cdot 4 \text{ ft} = 320 \text{ ft}^3.$$

Each bag holds 2 ft^3, so we needed $320 \div 2 = 160$ bags.

Example 6. The base of a solid is a right triangle with legs 3 cm and 4 cm. Find the volume of the prism and of the pyramid with this base, if the height of the prism is 9 cm and the height of the pyramid is 6 cm.



Solution. First we find the area of the base, a triangle:

$$B = \frac{4 \text{ cm} \cdot 3 \text{ cm}}{2} = 6 \text{ cm}^2.$$

For the prism,

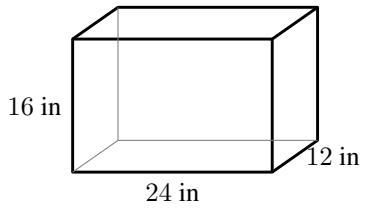
$$V = Bh = 6 \text{ cm}^2 \cdot 9 \text{ cm} = 54 \text{ cm}^3.$$

For the pyramid,

$$V = \frac{Bh}{3} = \frac{6 \text{ cm}^2 \cdot 6 \text{ cm}}{3} = 12 \text{ cm}^3.$$

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Example 7. A water tank is a rectangular prism 24 inches long, 12 inches wide, and 16 inches high. It is filled with water to a height of 12 inches. How many cubic inches of water are in the tank? How many more cubic inches does it take to fill the tank?



Solution. The base is a 24 by 12 rectangle, so $B = 24\,\text{in} \cdot 12\,\text{in} = 288\,\text{in}^2$. The water forms a prism of height 12 in:

$$V = 288\,\text{in}^2 \cdot 12\,\text{in} = 3456\,\text{in}^3.$$

The whole tank has volume $288 \cdot 16 = 4608\,\text{in}^3$, so it takes $4608 - 3456 = 1152\,\text{in}^3$ more water to fill it.

Example 8. A box is 2 feet long, 3 feet wide, and 1 foot high. Find its volume in cubic feet and in cubic inches.

Solution. In cubic feet,

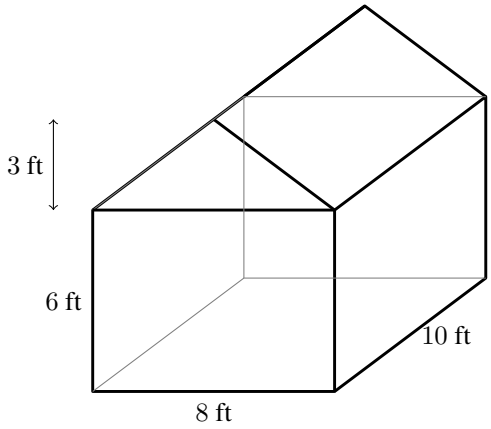
$$V = 2\,\text{ft} \cdot 3\,\text{ft} \cdot 1\,\text{ft} = 6\,\text{ft}^3.$$

For cubic inches, we first change each length to inches: the box is 24 in by 36 in by 12 in, so

$$V = 24\,\text{in} \cdot 36\,\text{in} \cdot 12\,\text{in} = 10368\,\text{in}^3.$$

Notice that we cannot simply multiply by 12: a cubic foot is a cube with edges 12 in, so $1\,\text{ft}^3 = 12 \cdot 12 \cdot 12 = 1728\,\text{in}^3$, and indeed $6 \cdot 1728 = 10368$.

Example 9. A small shed is a rectangular prism 8 feet wide, 10 feet long, and 6 feet tall, with a roof in the shape of a triangular prism that is 3 feet higher than the walls. Find the volume of the shed, including the roof space.



Solution. We find the volumes of the two parts and add them. The walls form a rectangular prism: $V = 8 \cdot 10 \cdot 6 = 480\,\text{ft}^3$. The roof is a prism whose base is a triangle with base 8 ft and height 3 ft, so $B = \dfrac{8 \cdot 3}{2} = 12\,\text{ft}^2$, and the height of this prism is the length of the shed, 10 ft: $V = 12 \cdot 10 = 120\,\text{ft}^3$. The total volume is

$$480 + 120 = 600\,\text{ft}^3.$$

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Example 10. A shipping crate is 6 feet long, 6 feet wide, and 8 feet high. How many boxes that are 2 feet by 3 feet by 4 feet fit into the crate?

Solution. The volume of the crate is $6 \cdot 6 \cdot 8 = 288\,\text{ft}^3$ and the volume of one box is $2 \cdot 3 \cdot 4 = 24\,\text{ft}^3$. If the boxes fill the crate completely with no gaps, we can fit

$$288 \div 24 = 12$$

boxes. (Dividing volumes tells us the most boxes that could possibly fit; here we can indeed stack 12 boxes: 3 along the length, 2 along the width, and 2 along the height.)

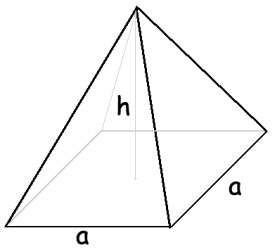


Practice Problems

Where an approximation is asked for, round to two decimal places.

1. What is the volume of a cube with edges $5\,\text{ft}$ long?

2. Find the volume of the square based pyramid shown on the picture, where $a = 4\,\text{cm}$ and $h = 6\,\text{cm}$.



3. Suppose that the volume of a rectangular prism is $24\,\text{m}^3$. How many such prisms are possible if all three sides are whole numbers? (Prisms with the same three sides in a different order are not counted separately.)

4. Find the volume of a box that is $5\,\text{in}$ long, $8\,\text{in}$ wide, and $3\,\text{in}$ high.

5. The base of a prism is a triangle with base $6\,\text{cm}$ and height $4\,\text{cm}$. The height of the prism is $9\,\text{cm}$. Find the volume of the prism, and of a pyramid with the same base and a height of $9\,\text{cm}$.

6. A box is 3 feet long, 2 feet wide, and 2 feet high. Find its volume in cubic feet and in cubic inches.

7. A swimming pool is 20 meters long, 10 meters wide, and 2 meters deep. How many cubic meters of water does it hold when full? How many cubic meters of water are in it when the water level is 0.5 meters below the top?

8. A solid is made of a rectangular prism with an $8\,\text{in}$ by $6\,\text{in}$ base and a height of $4\,\text{in}$, with a rectangular pyramid on top of it. The pyramid has the same base and a height of $3\,\text{in}$. Find the volume of the whole solid.

9. The volume of a rectangular pyramid is $70\,\text{cm}^3$. Its base is a rectangle $5\,\text{cm}$ by $7\,\text{cm}$. What is the height of the pyramid?

10. Every edge of a cube is doubled. How many times larger does the volume become? Check with a cube of edge 1 and a cube of edge 2.

11. Rectangular prism A is $3\,\text{in}$ long, $4\,\text{in}$ wide, and $5\,\text{in}$ high. Rectangular prism B is an enlargement of A by a factor of 2: each of its edges is twice as long. Find the volume of each prism. Is there a connection between the two volumes?

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12. Square based pyramid P has a base with sides of $3\,\mathrm{cm}$ and a height of $4\,\mathrm{cm}$. Pyramid Q is similar to P, with every edge twice as long: its base has sides of $6\,\mathrm{cm}$ and its height is $8\,\mathrm{cm}$. Find the volume of each pyramid. Is there a connection between the two volumes?

13. Find the volume of a cylinder with a base radius of $2\,\mathrm{cm}$ and a height of $7\,\mathrm{cm}$. Give the exact answer and an approximation rounded to two decimal places.

14. A cone has a base radius of $6\,\mathrm{in}$ and a height of $5\,\mathrm{in}$. Find its volume.

15. A can of soup is a cylinder with a diameter of $8\,\mathrm{cm}$ and a height of $10\,\mathrm{cm}$. How many cubic centimeters of soup does it hold when full?

16. A paper cup is shaped like a cone with a top radius of $3\,\mathrm{cm}$ and a height of $10\,\mathrm{cm}$. One cubic centimeter holds one milliliter. How many milliliters of water does the cup hold when full?

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Answers

1. $V = 5^3 = 125\,\text{ft}^3$.
2. $B = 4 \cdot 4 = 16\,\text{cm}^2$, $V = \dfrac{16 \cdot 6}{3} = 32\,\text{cm}^3$.
3. 6 possibilities: $(1, 1, 24)$, $(1, 2, 12)$, $(1, 3, 8)$, $(1, 4, 6)$, $(2, 2, 6)$, $(2, 3, 4)$.
4. $V = 5 \cdot 8 \cdot 3 = 120\,\text{in}^3$.
5. $B = \dfrac{6 \cdot 4}{2} = 12\,\text{cm}^2$. Prism: $V = 12 \cdot 9 = 108\,\text{cm}^3$. Pyramid: $V = \dfrac{12 \cdot 9}{3} = 36\,\text{cm}^3$.
6. $V = 3 \cdot 2 \cdot 2 = 12\,\text{ft}^3$. In inches the box is 36 by 24 by 24, so $V = 36 \cdot 24 \cdot 24 = 20736\,\text{in}^3$ (which is $12 \cdot 1728$).
7. Full: $20 \cdot 10 \cdot 2 = 400\,\text{m}^3$. With the water 1.5 meters deep: $20 \cdot 10 \cdot 1.5 = 300\,\text{m}^3$.
8. Prism: $48 \cdot 4 = 192\,\text{in}^3$. Pyramid: $\dfrac{48 \cdot 3}{3} = 48\,\text{in}^3$. Total: $240\,\text{in}^3$.
9. $B = 35\,\text{cm}^2$ and $70 = \dfrac{35h}{3}$, so $210 = 35h$ and $h = 6\,\text{cm}$.
10. $1^3 = 1$ and $2^3 = 8$: the volume becomes 8 times larger. (Doubling all three dimensions multiplies the volume by $2 \cdot 2 \cdot 2$.)
11. Prism A: $V = 3 \cdot 4 \cdot 5 = 60\,\text{in}^3$. Prism B is 6 in by 8 in by 10 in, so $V = 6 \cdot 8 \cdot 10 = 480\,\text{in}^3$. Since $480 = 8 \cdot 60$, the volume of the larger prism is 8 times the volume of the smaller one: each of the three dimensions doubled, and $2 \cdot 2 \cdot 2 = 8$.
12. Pyramid P: $B = 9\,\text{cm}^2$, $V = \dfrac{9 \cdot 4}{3} = 12\,\text{cm}^3$. Pyramid Q: $B = 36\,\text{cm}^2$, $V = \dfrac{36 \cdot 8}{3} = 96\,\text{cm}^3$. Since $96 = 8 \cdot 12$, the volume of the larger pyramid is again 8 times the volume of the smaller one.
13. $B = 4\pi$, $V = 28\pi\,\text{cm}^3 \approx 87.96\,\text{cm}^3$.
14. $B = 36\pi$, $V = \dfrac{36\pi \cdot 5}{3} = 60\pi\,\text{in}^3 \approx 188.50\,\text{in}^3$.
15. The radius is 4 cm, $B = 16\pi$, $V = 160\pi\,\text{cm}^3 \approx 502.65\,\text{cm}^3$.
16. $V = \dfrac{9\pi \cdot 10}{3} = 30\pi \approx 94.25\,\text{cm}^3$, so about 94.25 milliliters.

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