

The order of operations rule is an agreement among mathematicians, it simplifies notation.

P  
E  
M D  
A S

P stands for parentheses, E for exponents, M and D for multiplication and division, A and S for addition and subtraction. Notice that M and D are written next to each other. This is to suggest that multiplication and division are equally strong. Similarly, A and S are positioned to suggest that addition and subtraction are equally strong.

This is the hierarchy, and there are two basic rules.

1. **Between two operations that are on different levels of the hierarchy, we start with the operation that is higher.** For example, between a division and a subtraction, we start with the division since it is higher in the hierarchy than subtraction.
2. **Between two operations that are on the same level of the hierarchy, we start with the operation that comes first from reading left to right.**

These basic rules pretty much cover all possible situation. At every step, we execute only one operation, and replace the result by the expression indicating the operation. Let's see a few examples.

1.  $20 - 3 \cdot 4 =$

Solution: We observe two operations, a subtraction and a multiplication. Multiplication is higher in the hierarchy than subtraction, so we start there.

$$\begin{aligned} 20 - 3 \cdot 4 &= \text{multiplication} \\ 20 - 12 &= \text{subtraction} \\ &= 8 \end{aligned}$$

2.  $36 \div 3 \cdot 2 =$

Solution: it is a very common mistake to start with the multiplication. The letters M and D are in the same line because they are equally strong; among them we proceed left to right. From left to right, the division comes first

$$\begin{aligned} 36 \div 3 \cdot 2 &= \text{division} \\ 12 \cdot 2 &= \text{multiplication} \\ &= 24 \end{aligned}$$

3.  $36 \div 2 \div 2 =$

Solution: It is essential to perform these two divisions left to right. If we proceeded differently, we would get a different result.

$$\begin{aligned} 36 \div 2 \div 2 &= \text{first division from left} \\ 18 \div 2 &= \text{division} \\ &= 9 \end{aligned}$$