

Part 1. Sample Problems

Simplify each of the following expressions by applying the order of operations agreement.

1. $2 \cdot 3^2 - (6^2 - 2 \cdot 5) \div 2 = 5$

Solution: We start with the parentheses. We will work within the parentheses until the entire expression within it becomes one number. In the parentheses, there is an exponentiation, a subtraction, and a multiplication. Since it is stronger, we start with the exponent.

$$\begin{aligned} 2 \cdot 3^2 - (6^2 - 2 \cdot 5) \div 2 &= \text{exponent within parentheses} \\ 2 \cdot 3^2 - (36 - 2 \cdot 5) \div 2 &= \text{multiplication within parentheses} \\ 2 \cdot 3^2 - (36 - 10) \div 2 &= \text{subtraction within parentheses} \\ 2 \cdot 3^2 - (26) \div 2 &= \text{we may drop parentheses now} \\ 2 \cdot 3^2 - 26 \div 2 &= \end{aligned}$$

Now that there is no parentheses, we perform all exponents, left to right. There is only one, so we have

$$\begin{aligned} 2 \cdot 3^2 - 26 \div 2 &= \text{exponent} \\ 2 \cdot 9 - 26 \div 2 &= \end{aligned}$$

Now we execute all multiplications, divisions, left to right

$$\begin{aligned} 2 \cdot 9 - 26 \div 2 &= \text{multiplication} \\ 18 - 26 \div 2 &= \text{division} \\ 18 - 13 &= \text{subtraction} \\ &= 5 \end{aligned}$$

2. $18 - 7 - 3 = 8$

Solution: It is a common mistake to subtract 4 from 18. This is not what order of operations tell us to do. The two subtractions have to be performed left to right.

$$\begin{aligned} 18 - 7 - 3 &= \text{first subtraction from left} \\ 11 - 3 &= \text{subtraction} \\ &= 8 \end{aligned}$$

3. $5^2 - 2(10 - 2^2) = 13$

Solution: We start with the parentheses

$$\begin{aligned} 5^2 - 2(10 - 2^2) &= \text{exponent in parentheses} \\ 5^2 - 2(10 - 4) &= \text{subtraction in parentheses} \\ 5^2 - 2(6) &= \text{drop parentheses} \\ 5^2 - 2 \cdot 6 &= \text{exponents} \\ 25 - 2 \cdot 6 &= \text{multiplication} \\ 25 - 12 &= \text{subtraction} \\ &= 13 \end{aligned}$$

4. $8^2 - 3^2 = 55$

Solution: There are three operations, two exponents and a subtraction. We start with the exponents, left to right.

$$\begin{aligned} 8^2 - 3^2 &= \text{first exponent from left} \\ 64 - 3^2 &= \text{exponent} \\ 64 - 9 &= \text{subtraction} \\ &= 55 \end{aligned}$$

5. $(8 - 3)^2 = 25$

Solution: We start with the parentheses

$$\begin{aligned} (8 - 3)^2 &= \text{subtraction in parentheses} \\ (5)^2 &= \text{drop parentheses} \\ 5^2 &= \text{exponents} \\ &= 25 \end{aligned}$$

This problem and the previous one tells us a very important thing: $a^2 - b^2$ and $(a - b)^2$ are different expressions! In $a^2 - b^2$ we first square a and b and then subtract. In $(a - b)^2$ we first subtract b from a and then square the difference.

6. $(3^3 - 4 \cdot 5 + 2)^2 = 81$

Solution: We will work within the parentheses until it becomes a number. Within the parentheses, we start with the exponents.

$$\begin{aligned} (3^3 - 4 \cdot 5 + 2)^2 &= \text{exponents within parentheses} \\ (27 - 4 \cdot 5 + 2)^2 &= \text{multiplication within parentheses} \\ (27 - 20 + 2)^2 &= \end{aligned}$$

There is an addition and a subtraction in the parentheses. **It is not true that addition comes before subtraction!** Addition and subtraction are equally strong; we execute them left to right.

$$\begin{aligned} (27 - 20 + 2)^2 &= \text{subtraction within parentheses} \\ (7 + 2)^2 &= \text{addition within parentheses} \\ (9)^2 &= \text{drop parentheses} \\ 9^2 &= \text{exponents} \\ &= 81 \end{aligned}$$

$$7. \frac{3 + 2(20 - 3^2 - 5)}{3^2 - 2^2} + 4^1 = 7$$

Solution: The division bar stretching over entire expressions is a case of the **invisible parentheses**. It instructs us to work out the top until we obtain a number, the bottom until we obtain a number, and finally divide. The invisible parentheses here means

$$\frac{3 + 2(20 - 3^2 - 5)}{3^2 - 2^2} = [3 + 2(20 - 3^2 - 5)] \div [3^2 - 2^2]$$

And now we see that the invisible parentheses was developed to simplify notation. We will start with the top. Naturally, we stay within the parentheses until they disappear.

$$\begin{aligned} \frac{3 + 2(20 - 3^2 - 5)}{3^2 - 2^2} + 4^1 &= \text{exponent in parentheses} \\ \frac{3 + 2(20 - 9 - 5)}{3^2 - 2^2} + 4^1 &= \text{first subtraction from left in parentheses} \\ \frac{3 + 2(11 - 5)}{3^2 - 2^2} + 4^1 &= \text{subtraction in parentheses} \\ \frac{3 + 2(6)}{3^2 - 2^2} + 4^1 &= \text{drop parentheses} \\ \frac{3 + 2 \cdot 6}{3^2 - 2^2} + 4^1 &= \text{multiplication on top} \\ \frac{3 + 12}{3^2 - 2^2} + 4^1 &= \text{addition on top} \\ \frac{15}{3^2 - 2^2} + 4^1 &= \end{aligned}$$

Now we work out the bottom, applying order of operations

$$\begin{aligned} \frac{15}{3^2 - 2^2} + 4^1 &= \text{first exponent from left to right} \\ \frac{15}{9 - 2^2} + 4^1 &= \text{exponent} \\ \frac{15}{9 - 4} + 4^1 &= \text{subtraction} \\ \frac{15}{5} + 4^1 &= \text{same as} \\ 15 \div 5 + 4^1 &= \end{aligned}$$

We now have a division, an addition, and an exponent. We start with the exponent.

$$\begin{aligned} 15 \div 5 + 4^1 &= \text{exponent, } 4^1 = 4 \\ 15 \div 5 + 4 &= \text{division} \\ 3 + 4 &= \text{addition} \\ &= 7 \end{aligned}$$

Part 2. Practice Problems

1. $2 \cdot 5^2 - (6 \cdot 5 - 3^2) \div 3 = 43$

2. $10^2 - 7^2 = 51$

3. $(10 - 7)^2 = 9$

4. $20 - 7 - 1 = 12$

5. $2^3 - (11 - 3^2)^2 = 4$

6. $\frac{5^2 - 3^2}{2^2} = 4$

7. $\frac{(5 - 3)^2}{2^2} = 1$

8. $120 \div 6 \cdot 2 = 40$

9. $((7 - 4)^2 - 5)^2 - 1 = 15$

10. $\frac{22 - 3^2 + 2(20 - 3^2 - 5)}{3^2 - 2^2} = 5$

11. $\frac{5 + (5^2 - 3^2)}{3^2 - 2 \cdot 1^8} = 3$

12. $30 - (2(15 - 2^3) - 2^2) = 20$

13. $4(3(2(2^2 - 1) + 1) - 1) + 5 = 85$

14. $\frac{2(3^3 - 4 \cdot 5) - 2^2}{4^2 - (3^2 + 2)} = 2$