

1. Find all critical points of $f(x, y) = x^2 + y^2 - 2x - 6y + 14$.
2. Find the minimal distance from the origin to the plane $x - y + 2z = 3$.
3. Find the local maximum or minimum values of $f(x, y) = x^2 - 2x + \frac{y^2}{4}$.
4. Find local extrema for $z = f(x, y) = -x^2 - y^2 + 6x + 8y - 21$.
5. a) Apply the maximum-minimum test to $f(x, y) = x^2 + 3xy + y^2 + 16$.
 b) Determine whether $(0, 0)$ is a maximum-point, a minimum-point, or neither for $g(x, y) = 3x^2 - 5xy + 3y^2$.
6. Find the critical points of each function and classify them as local maxima, minima, or neither.
 - a) $f(x, y) = -\frac{x^2}{a^2} + \frac{y^2}{b^2}$
 - c) $f(x, y) = e^{(1-x^2-y^2)}$
 - b) $f(x, y) = x^4 + y^4 - 4xy + 1$
 - d) $f(x, y) = (x^2 - y^2)e^{\frac{-x^2-y^2}{2}}$
7. Find the shortest distance from $(1, 0, -2)$ to the plane $x + 2y + z = 4$.
8. An open rectangular box is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.
9. A surfboard company has developed the yearly profit equation

$$P(x, y) = -22x^2 + 22xy - 11y^2 + 110x - 44y - 23$$

where x is the number (in thousands) of standard surfboards produced per year, and y is the number (in thousands) of competition surfboards produced each year, and P is the profit (in thousands of dollars). How many of each type of board should be produced per year to realize a maximal profit? What is the maximal profit?

10. A department store sells two brands of inexpensive calculators. The store pays \$6 for each brand A calculator and \$8 for each brand B calculator. The research department has estimated the following weekly demand for these two competitive products.

$$x = 116 - 30p + 20q \quad \text{and} \quad y = 144 + 16p - 24q$$

where p is the selling price for brand A and q is the selling price for brand B .

- a) Determine the demands x and y when $p = \$10$ and $q = \$12$, when $p = q = \$11$.
- b) How should the store price each calculator to maximize weekly profit? What is the maximum weekly profit?

Answers

- 1.) $(1, 3, 4)$. 2.) closest point: $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$, min distance: $\frac{\sqrt{6}}{2}$ 3.) $(1, 0, -1)$ is a global min
- 4.) $(3, 4, 4)$ is a local and global maximum 5.) a) $(0, 0, 16)$ is a saddle point. b) min at $(0, 0)$
- 6.) a) $(0, 0, 0)$ is a saddle point
- b) $(0, 0, 1)$ is a saddle point, and $(1, 1, -1)$ and $(-1, -1, -1)$ are local minimums. c) local max: $(0, 0, e)$
- d) f has a saddle point at $(0, 0)$, a max at $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$, and a min at $(0, \sqrt{2})$ and $(0, -\sqrt{2})$.
- 7.) $\frac{5}{6}\sqrt{6}$ 8.) 4 m^3
- 9.) max pr: \$120 000 when 3000 standard and 1000 competition boards are produced.
- 10.) a) when $p = \$10$ and $q = \$12$, then $x = 56$ and $y = 16$ when $p = q = \$11$. then $x = 6$ and $y = 56$
- b) $p = 10, q = 12$ then the max profit is 288