

- Find all critical points of $f(x, y) = x^2 + y^2 - 2x - 6y + 14$. $(1, 3, 4)$.
- Find the minimal distance from the origin to the plane $x - y + 2z = 3$. closest point:
 $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$, min distance: $\frac{\sqrt{6}}{2}$.
- Find the local maximum or minimum values of $f(x, y) = x^2 - 2x + \frac{y^2}{4}$. $(1, 0, -1)$ is a global min
- Find local extrema for $z = f(x, y) = -x^2 - y^2 + 6x + 8y - 21$. $(3, 4, 4)$ is a local and global maximum
- a) Apply the maximum-minimum test to $f(x, y) = x^2 + 3xy + y^2 + 16$ $(0, 0, 16)$ is a saddle point.
 b) Determine whether $(0, 0)$ is a maximum-point, a minimum-point, or neither for $g(x, y) = 3x^2 - 5xy + 3y^2$. min at $(0, 0)$
- Find the critical points of each function and classify them as local maxima, minima, or neither.
 - $f(x, y) = -\frac{x^2}{a^2} + \frac{y^2}{b^2}$
 - $f(x, y) = x^4 + y^4 - 4xy + 1$
 - $f(x, y) = e^{(1-x^2-y^2)}$
 - $f(x, y) = (x^2 - y^2)e^{\frac{-x^2-y^2}{2}}$

Solution:

- $f(x, y) = -\frac{x^2}{a^2} + \frac{y^2}{b^2}$ $(0, 0, 0)$ is a saddle point
- $f(x, y) = x^4 + y^4 - 4xy + 1$

critical point	A	B	C	$AC - B^2$	Type
$(0, 0)$	0	-4	0	-16	saddle point
$(1, 1)$	12	-4	12	128	minimum
$(-1, -1)$	12	-4	12	128	minimum

Thus $(0, 0, 1)$ is a saddle point, and $(1, 1, -1)$ and $(-1, -1, -1)$ are local minima.

- $f(x, y) = e^{(1-x^2-y^2)}$ local max: $(0, 0, e)$
- $f(x, y) = (x^2 - y^2)e^{\frac{-x^2-y^2}{2}}$

critical point	A	B	C	$AC - B^2$	Type
$(0, 0, 0)$	2	0	-2	-4	saddle point
$(\sqrt{2}, 0, 2/e)$	$-4/e$	0	$-4/e$	$16/e^2$	maximum
$(-\sqrt{2}, 0, 2/e)$	$-4/e$	0	$-4/e$	$16/e^2$	maximum
$(0, \sqrt{2}, -2/e)$	$4/e$	0	$4/e$	$16/e^2$	minimum
$(0, -\sqrt{2}, -2/e)$	$4/e$	0	$4/e$	$16/e^2$	minimum

- Find the shortest distance from $(1, 0, -2)$ to the plane $x + 2y + z = 4$. $\frac{5}{6}\sqrt{6}$.

8. An open rectangular box is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box. 4 m^3 .
9. A surfboard company has developed the yearly profit equation

$$P(x, y) = -22x^2 + 22xy - 11y^2 + 110x - 44y - 23$$

where x is the number (in thousands) of standard surfboards produced per year, and y is the number (in thousands) of competition surfboards produced each year, and P is the profit (in thousands of dollars). How many of each type of board should be produced per year to realize a maximum profit? What is the maximal profit?

max pr: \$120 000 when 3000 standard and 1000 competition boards are produced.

10. A department store sells two brands of inexpensive calculators. The store pays \$6 for each brand A calculator and \$8 for each brand B calculator. The research department has estimated the following weekly demand for these two competitive products.

$$x = 116 - 30p + 20q \quad \text{and} \quad y = 144 + 16p - 24q$$

where p is the selling price for brand A and q is the selling price for brand B .

a) Determine the demands x and y when $p = \$10$ and $q = \$12$, when $p = q = \$11$. $(56, 16)$
 $(6, 56)$

b) How should the store price each calculator to maximize weekly profit? What is the maximum weekly profit? $P(10, 12) = 288$