

Sample Problems

1. Consider the function $f(x) = -2x^3 + 9x^2 + 24$.
 - a) Find all values of x for which f is increasing.
 - b) Find all values of x for which f is decreasing.
 - c) Find all values of x for which f has a relative maximum.
 - d) Find all values of x for which f has a relative minimum.
 - e) Sketch the graph of f .

2. Consider the function $f(x) = 6x^5 - 50x^3 - 120$.
 - a) Find all values of x for which f is increasing.
 - b) Find all values of x for which f is decreasing.
 - c) Find all values of x for which f has a relative maximum.
 - d) Find all values of x for which f has a relative minimum.
 - e) Sketch the graph of f .

Practice Problems

For each of the following functions f given below,

- a) Find all values of x for which f is increasing.
- b) Find all values of x for which f is decreasing.
- c) Find all values of x for which f has a relative maximum.
- d) Find all values of x for which f has a relative minimum.
- e) Sketch the graph of f .

1.) $f(x) = x^3 - 3x^2 + 6$

5.) $f(x) = -6x^5 + 20x^3$

2.) $f(x) = -x^3 + 6x^2 + 36x - 60$

6.) $f(x) = 5x^6 - 24x^5 + 30x^4 - 12$

3.) $f(x) = -2x^3 + 12x^2 + 6x - 24$

7.) $f(x) = -3x^4 + 16x^3 + 1$

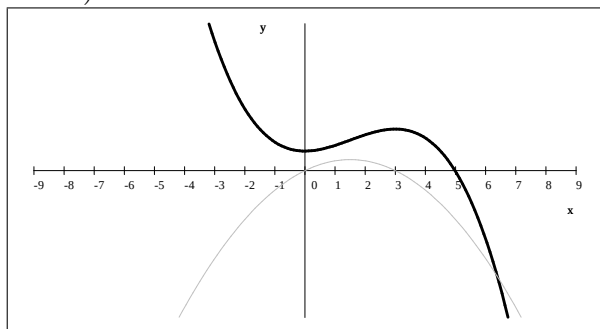
4.) $f(x) = 3x^4 - 6x^2 + 8$

8.) $f(x) = -x^6 + 6x^4$

Sample Problems - Answers

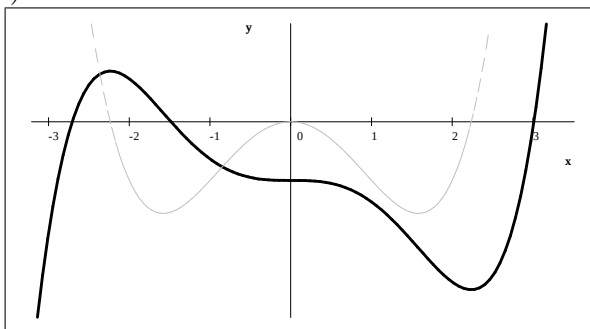
1.) $f(x) = -2x^3 + 9x^2 + 24.$

- a) increasing on $(0, 3)$
- b) decreasing on $(-\infty, 0)$ and on $(3, \infty)$
- c) relative maximum at $x = 3$
- d) relative minimum at $x = 0$
- e)



2.) $f(x) = 6x^5 - 50x^3 - 120$

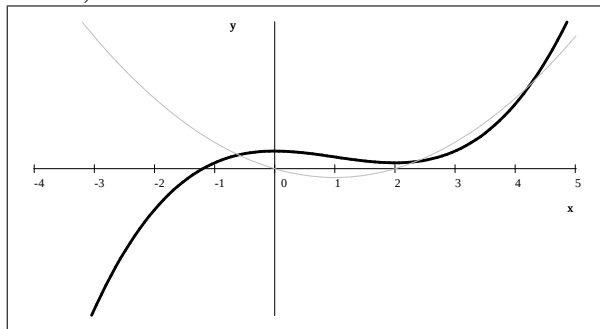
- a) increasing on $(-\infty, -\sqrt{5})$ and on $(\sqrt{5}, \infty)$
- b) decreasing on $(-\sqrt{5}, \sqrt{5})$
- c) relative maximum at $x = -\sqrt{5}$
- d) relative minimum at $x = \sqrt{5}$
- e)



Practice Problems - Answers

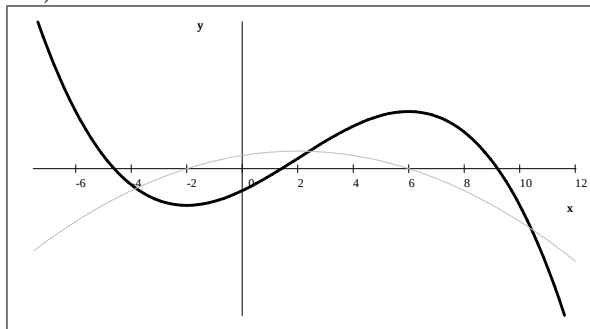
1.) $f(x) = x^3 - 3x^2 + 6$

- a) increasing on $(-\infty, 0)$ and on $(2, \infty)$
- b) decreasing on $(0, 2)$
- c) relative maximum at $x = 0$
- d) relative minimum at $x = 2$
- e)



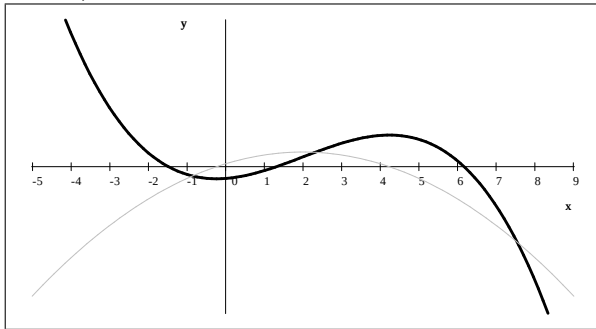
2.) $f(x) = -x^3 + 6x^2 + 36x - 60$

- a) increasing on $(-2, 6)$
- b) decreasing on $(-\infty, -2)$ and on $(6, \infty)$
- c) relative maximum at $x = 6$
- d) relative minimum at $x = -2$
- e)



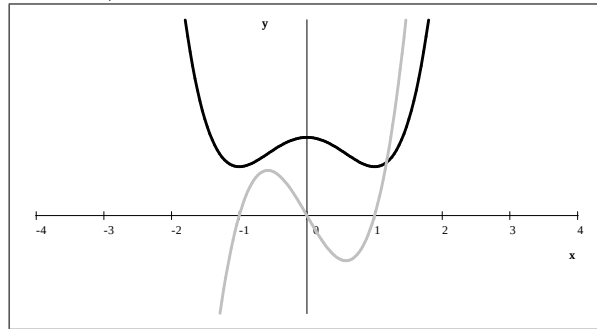
3.) $f(x) = -2x^3 + 12x^2 + 6x - 24$

- a) increasing on $(2 - \sqrt{5}, 2 + \sqrt{5})$
- b) decreasing on $(-\infty, 2 - \sqrt{5})$ and on $(2 + \sqrt{5}, \infty)$
- c) relative maximum at $x = 2 + \sqrt{5}$
- d) relative minimum at $x = 2 - \sqrt{5}$
- e)



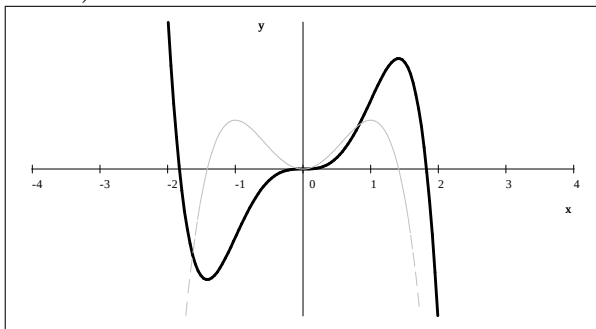
4.) $f(x) = 3x^4 - 6x^2 + 8$

- a) increasing on $(-1, 0)$ and on $(1, \infty)$
- b) decreasing on $(-\infty, -1)$ and on $(0, 1)$
- c) relative maximum at $x = 0$
- d) relative minimum at $x = -1, 1$
- e)



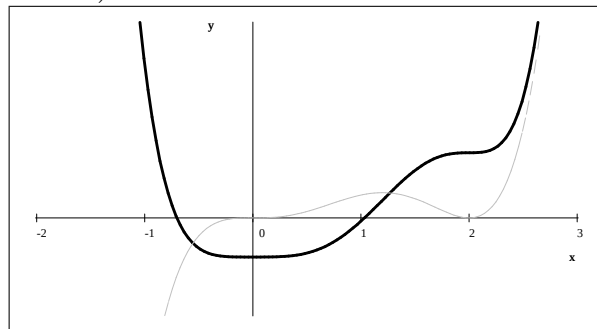
5.) $f(x) = -6x^5 + 20x^3$

- a) increasing on $(-\sqrt{2}, 0)$ and on $(0, \sqrt{2})$
- b) decreasing on $(-\infty, -\sqrt{2})$ and on $(\sqrt{2}, \infty)$
- c) relative maximum at $x = \sqrt{2}$
- d) relative minimum at $x = -\sqrt{2}$
- e)



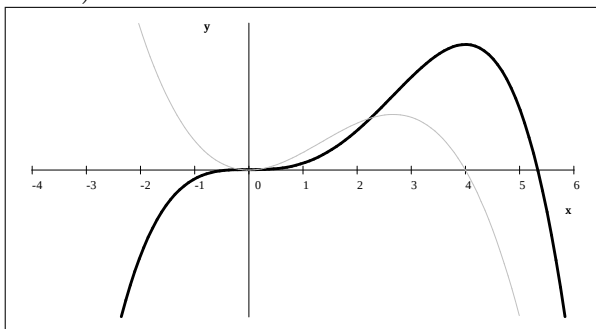
6.) $f(x) = 5x^6 - 24x^5 + 30x^4 - 12$

- a) increasing on $(0, \infty)$
- b) decreasing on $(-\infty, 0)$
- c) no relative maximum
- d) relative minimum at $x = 0$
- e)



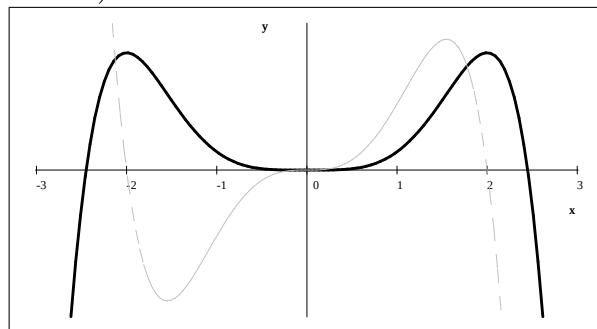
7.) $f(x) = -3x^4 + 16x^3 + 1$

- a) increasing on $(-\infty, 4)$
- b) decreasing on $(4, \infty)$
- c) relative maximum at $x = 4$
- d) no relative minimum
- e)



8.) $f(x) = -x^6 + 6x^4$

- a) increasing on $(-\infty, -2)$ and on $(0, 2)$
- b) decreasing on $(-2, 0)$ and on $(2, \infty)$
- c) relative maximum at $x = -2, 2$
- d) relative minimum at $x = 0$
- e)



Sample Problems - Solutions

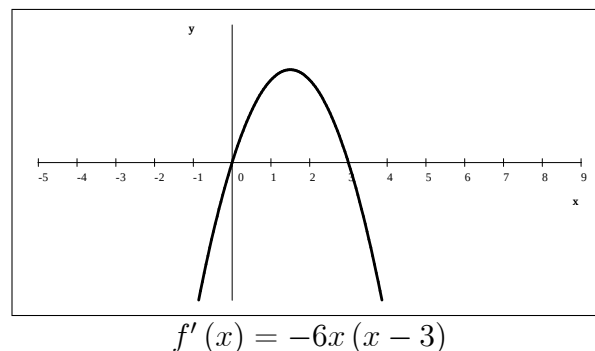
1.) Consider the function $f(x) = -2x^3 + 9x^2 + 24$.

a) Find all values of x for which f is increasing.

Solution: We compute f' first and then determine when f' is positive.

$$\begin{aligned}f(x) &= -2x^3 + 9x^2 + 24 \\f'(x) &= -6x^2 + 18x = -6x(x - 3)\end{aligned}$$

f' is a quadratic function with a negative leading coefficient. Thus its graph is a downward opening parabola. The x -intercepts are at $x = 0$ and $x = 3$. The graph of f' is thus



When f' is positive, then f is increasing. We can see that f' is positive on $(0, 3)$ and so f is increasing there. The answer is: f is increasing on $(0, 3)$.

b) Find all values of x for which f is decreasing.

Solution: we differentiate f , and then factor and graph f' (see in part a). When f' is negative, then f is decreasing. From the graph we can see that f' is negative on $(-\infty, 0)$ and $(3, \infty)$ and so f is decreasing there. The answer is: f is decreasing on $(-\infty, 0)$ and on $(3, \infty)$.

c) Find all values of x for which f has a relative maximum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative maximum at x if f changes from increasing to decreasing at x . That happens when f' changes sign from positive to negative. Based on the answers for parts a) and b), this happens at $x = 3$. Thus f has a relative maximum at $x = 3$.

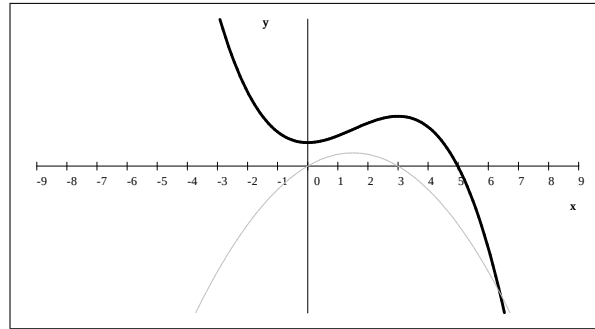
d) Find all values of x for which f has a relative minimum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative minimum at x if f changes from decreasing to increasing at x . That is the same as f' changing from negative to positive. Based on the answers for parts a) and b), this happens at $x = 0$. Thus f has a relative minimum at $x = 0$.

e) Sketch the graph of f .

Solution: We evaluate f at the relative minimum and maximum.

$f(0) = 24$ and $f(3) = 51$. Note that the only x -intercept of f is irrational and it would take solving a cubic equation to find its exact value. If needed, we may evaluate f at additional points. In case of polynomials, the y -intercept is a very easy one.



2.) Consider the function $f(x) = 6x^5 - 50x^3 - 120$.

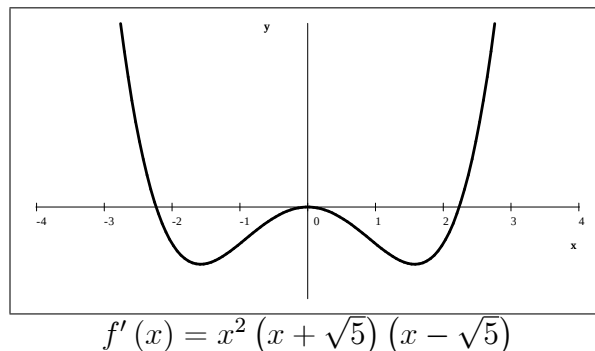
a) Find all values of x for which f is increasing.

Solution: Let us compute f' first.

$$f(x) = 6x^5 - 50x^3 - 120$$

$$f'(x) = 30x^4 - 150x^2 = 30x^2(x^2 - 5) = 30x^2(x + \sqrt{5})(x - \sqrt{5})$$

f' is a degree four polynomial function with a positive leading coefficient and x -intercepts at $x = -\sqrt{5}, 0, \sqrt{5}$. The graph of f' is thus



When f' is positive, then f is increasing. We can see that f' is positive on $(-\infty, -\sqrt{5})$ and $(\sqrt{5}, \infty)$ so f is increasing there. The answer is: f is increasing on $(-\infty, -\sqrt{5})$ and on $(\sqrt{5}, \infty)$.

b) Find all values of x for which f is decreasing.

Solution: we differentiate f , and then factor and graph f' (see in part a). When f' is negative, then f is decreasing. From the graph we can see that f' is negative on $(-\sqrt{5}, 0)$ and on $(0, \sqrt{5})$ and so f is decreasing there.

In this particular case, f is decreasing on the entire interval $(-\sqrt{5}, \sqrt{5})$. One must be careful when making this step. For example, the function $g(x) = \frac{1}{x}$ is decreasing on $(-\infty, 0)$ and $(0, \infty)$ but is not decreasing on $(-\infty, \infty)$.

- c) Find all values of x for which f has a relative maximum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative maximum at x if f changes from increasing to decreasing at x . That happens when f' changes sign from positive to negative. Based on the answers for parts a) and b), this happens at $x = -\sqrt{5}$. Thus f has a relative maximum at $x = -\sqrt{5}$.

- d) Find all values of x for which f has a relative minimum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative minimum at x if f changes from decreasing to increasing at x . That happens when f' changes sign from negative to positive. Based on the answers for parts a) and b), this happens at $x = \sqrt{5}$. Thus f has a relative minimum at $x = \sqrt{5}$.

What about the zero of f' at $x = 0$? Does f have a relative maximum or a minimum at $x = 0$? The answer is: neither. The derivative f' is negative on an interval before $x = 0$ and also on an interval after $x = 0$. Consequently, f is decreasing before and after $x = 0$ and so there can be neither a relative maximum nor a relative minimum there. This situation is similar to $g(x) = x^3$ at $x = 0$. Although its derivative, $g'(x) = 3x^2$ has a zero at $x = 0$, there is no change in sign around the zero there, and so $g(x) = x^3$ does not have a relative minimum or maximum at $x = 0$.

- e) Sketch the graph of f .

Solution: We evaluate f at the relative minimum and maximum.

$f(0) = -120$, $f(-\sqrt{5}) = -120 + 100\sqrt{5} \approx 103.6068$ and $f(\sqrt{5}) = -120 - 100\sqrt{5} \approx -343.6068$.

