

- Simplify:
 - $\frac{x^3 - 9x}{x^2 - 7x + 12}$
 - $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$
 - $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$
 - $\frac{9 - x^{-2}}{3 + x^{-1}}$
 - $\frac{x(x+3)}{x-4}$
 - $\frac{x-4}{x(x-1)}$
 - $\frac{5x}{x+5}$
 - $\frac{3x-1}{x}$
- Rationalize the denominator:
 - $\frac{2}{\sqrt{3} + \sqrt{2}}$
 - $\frac{4}{1 - \sqrt{5}}$
 - $2(\sqrt{3} - \sqrt{2})$
 - $-1 - \sqrt{5}$
- Assume that all variables represent positive numbers. Write each of the following expressions in the form ca^pb^q where c, p, q are numbers:
 - $\frac{(2a^2)^3}{b}$
 - $\sqrt{9ab^3}$
 - $\frac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$
 - $\frac{ab-a}{b^2-b}$
 - $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$
 - $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$
 - $8a^6b^{-1}$
 - $3a^{\frac{1}{2}}b^{\frac{3}{2}}$
 - $\frac{2}{3}a^2b^{-1}$
 - ab^{-1}
 - $a^{-\frac{3}{2}}b$
 - $a^{\frac{5}{6}}b^{\frac{1}{2}}$
- Solve for x . Do not use a calculator.
 - $5^{x+1} = 25$
 - $\frac{1}{3} = 3^{2x+2}$
 - $\log_2 x = 3$
 - $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$
 - 1
 - $-\frac{3}{2}$
 - 8
 - $\pm\frac{4}{25}$
- Simplify
 - $\log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$
 - $3^{2\log_3 5}$
 - $2\log_4 9 - \log_2 3$
 - $\log_2 (5(x+1))$
 - 25
 - $\log_2 3$
- Simplify
 - $\log_{10} (10^{1/2})$
 - $\log_{10} \left(\frac{1}{10^x}\right)$
 - $2\log_{10} \sqrt{x} + 3\log_{10} x^{1/3}$
 - $\frac{1}{2}$
 - $-x$
 - $2\log_{10} x$
- Solve the following equations for the indicated variables.
 - $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, for a
 - $S = 2(ab + bc + ac)$, for a
 - $A = 2\pi r^2 + 2\pi rh$, for positive r
 - $A = P + nPr$, for P
 - $2x - 2yd = y + xd$, for d
 - $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ for x
 - $\frac{bcx}{bc - cy - bz}$
 - $\frac{S - 2bc}{2(b+c)}$
 - $\frac{-\pi h + \sqrt{\pi^2 h^2 + 2A\pi}}{2\pi}$
 - $\frac{A}{nr+1}$
 - $\frac{2x-y}{x+2y}$
 - $\frac{\pi}{\pi-1}$
- Find the vertex for each of the following parabolas.
 - $y = x^2 + 4x + 3$
 - $3x^2 + 3x + 2y = 0$
 - $9y^2 - 6y - 9 - x = 0$
 - $(-2, -1)$
 - $\left(-\frac{1}{2}, \frac{3}{8}\right)$
 - $\left(-10, \frac{1}{3}\right)$
- Find all real solutions to each of the following equations.
 - $x^6 - 16x^4 = 0$
 - $4x^3 - 8x^2 - 25x + 50 = 0$
 - $8x^3 + 27 = 0$
 - $x^4 = 1$
 - $-4, 0, 4$
 - $-\frac{5}{2}, \frac{5}{2}, 2$
 - $-\frac{3}{2}$
 - $-1, 1$

10. Solve the equations: a) $4x^2 + 12x + 3 = 0$ b) $2x + 1 = \frac{5}{x+2}$ c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

a) $\frac{-3 \pm \sqrt{6}}{2}$ b) $-3, \frac{1}{2}$ c) $-\frac{1}{2}$

11. Find the remainders on division of

a) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$ b) $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

a) -89 b) $x^2 + 3$

12. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

$-\frac{1}{3}, \frac{1}{4}, 2$

13. Solve the inequalities a) $x^2 + 2x - 3 \leq 0$ b) $\frac{2x-1}{3x-2} \leq 1$ c) $x^2 + x + 1 > 0$

a) $-3 \leq x \leq 1$ b) $x < \frac{2}{3}$ or $x > 1$ c) all x

14. Solve for x : a) $|5x - 2| = 8$ b) $|2x + 1| = |x + 3|$

a) $-\frac{6}{5}, 2$ b) $-\frac{4}{3}, 2$

15. Determine the equations of the following lines:

a) the line through $(-1, 3)$ and $(2, -4)$

b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$

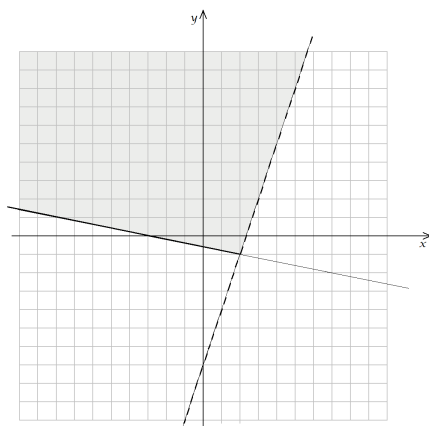
c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

a) $7x + 3y = 2$ b) $3x + 2y = 1$ c) $y = 3$

16. a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

b) Shade the region in the xy -plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$.

a) $(2, -1)$ b)



17. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find

a) the center and radius b) the equation of the tangent line at $(-2, 5)$

a) $C(-3, 2)$ $r = \sqrt{10}$ b) $x + 3y = 13$

18. Find the equations of the following circles:

- a) the circle with center at $(1, 2)$ that passes through the point $(-2, -1)$
 b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the x - and y -axes, respectively.
 c) a circle is tangent to the y -axis at $y = 3$ and has one x -intercept at $x = 1$.

a) $(x - 1)^2 + (y - 2)^2 = 18$ b) $\left(x - \frac{1}{2}\right)^2 + (y - 1)^2 = \frac{5}{4}$ c) $(x - 5)^2 + (y - 3)^2 = 25$

19. a) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \frac{5x - 3}{2x + 1}$

b) Find the domain of the function $f(x) = \frac{3x + 1}{\sqrt{x^2 + x - 2}}$

a) i) D : all real numbers $R = \{7\}$

ii) D : all real numbers except $-\frac{1}{2}$ R : all real numbers except $\frac{5}{2}$

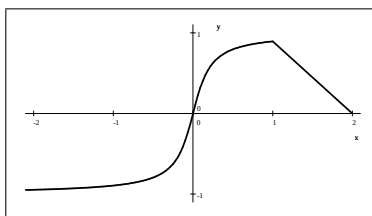
b) $x < -2$ or $x > 1$

20. Simplify $\frac{f(x + h) - f(x)}{h}$ where

a) $f(x) = 2x + 3$ b) $f(x) = x^2$ c) $f(x) = \frac{1}{x + 1}$

a) 2 b) $h + 2x$ c) $\frac{-1}{(x + 1)(x + h + 1)}$

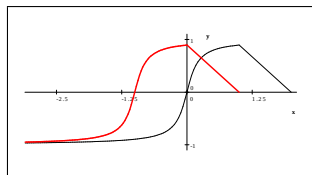
21. The graph of a function $y = f(x)$ is given as follows:



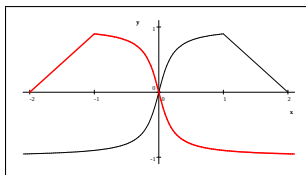
Plot the graph of each of the following functions:

a) $f(x + 1)$ b) $f(-x)$ c) $|f(x)|$ d) $f(|x|)$

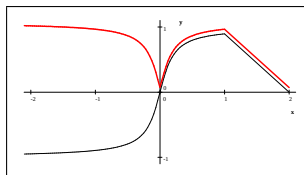
a)



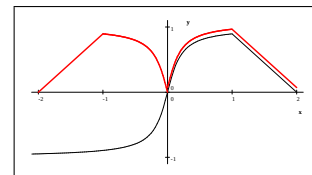
b)



c)



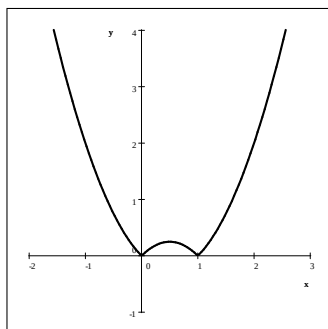
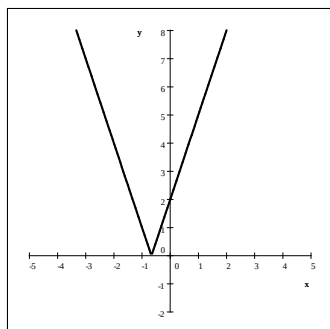
d)



22. Sketch the graph of the functions: a) $g(x) = |3x + 2|$ b) $h(x) = |x(x - 1)|$

a)

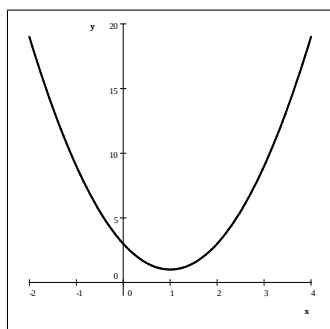
b)



23. a) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$
 b) The graph of a quadratic function (a parabola) has x -intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4 . Determine an expression for the function.

a)

b) $y = -x^2 + 2x + 3$

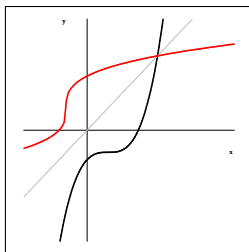
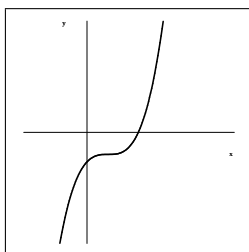


24. Find the inverse of the following functions.

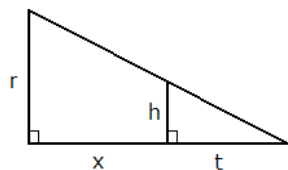
a) $f(x) = 2x + 3$ b) $f(x) = \frac{x+2}{5x-1}$ c) $f(x) = x^2 + 2x - 1, \quad x > 0$

a) $f^{-1}(x) = \frac{x-3}{2}$ b) $f^{-1}(x) = \frac{x+2}{5x-1}$ c) $f^{-1}(x) = \sqrt{x+2} - 1, \quad x > -1$

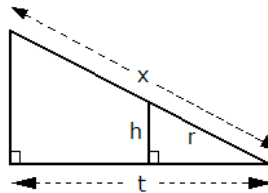
25. A function $f(x)$ has the following graph. Sketch the graph of the inverse function $f^{-1}(x)$.



26. Express x in terms of the other variables in the picture.



a)



b)

a) $x = t \left(\frac{r-h}{h} \right)$ b) $\frac{rt}{\sqrt{r^2 - h^2}}$

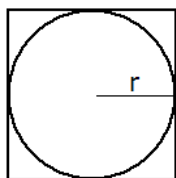
27. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 2)$ is twice its distance from the point $B(2, -1)$. Determine the equation of the curve.

$x^2 - 6x + y^2 + 4y + 5 = 0$ (a circle)

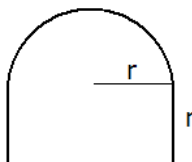
28. Let $f(x) = \frac{|x|}{x}$. Show that $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$. Find the domain and range of $f(x)$.

D : all numbers except 0; $R = \{-1, 1\}$

29. a) Find the ratio of the area inside the square but outside the circle to the area of the square in picture (a) below.



a)



b)

- b) Find a formula for the perimeter of a window of the shape on picture (b) above.

- c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

- d) Two cars start moving from the same point. One travels south at $100 \frac{\text{km}}{\text{h}}$ (kilometer per hour), the other west at $50 \frac{\text{km}}{\text{h}}$. How far apart are they two hours later?

- e) A kite is 100m above the ground. If there is 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

a) $1 - \frac{\pi}{4}$ b) $4r + \pi r$ c) $\frac{9\pi}{4} \text{ m}^2$ d) $100\sqrt{5} \text{ km}$ e) $\frac{\pi}{6}$ or 30°

30. Without using a calculator, evaluate each of the following:

a) $\cos 210^\circ$ b) $\sin \frac{5\pi}{3}$ c) $\tan^{-1}(-1)$ d) $\sin^{-1}(-1)$

e) $\cos \frac{9\pi}{4}$ f) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ g) $\tan\left(\frac{7\pi}{6}\right)$ h) $\cos^{-1}(-1)$

a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $-\frac{\pi}{4}$ d) $-\frac{\pi}{2}$ e) $\frac{\sqrt{2}}{2}$ f) $\frac{\pi}{3}$ g) $\frac{\sqrt{3}}{3}$ h) π

31. Solve for x : a) $3 \sin^2 x = \cos^2 x$; $0 \leq x \leq 2\pi$ b) $\cos^2 x - \sin^2 x = \sin x$; $-\pi \leq x \leq \pi$
 c) $\tan x + \sec x = 2 \cos x$; $-\infty < x < \infty$

a) $3 \sin^2 x = \cos^2 x$, Solution is: $-\frac{1}{6}\pi + \pi k$

a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ c) $\frac{\pi}{6} + 2k\pi$ or $\frac{5\pi}{6} + 2k\pi$ where k is any integer

32. Assume the following. For all real numbers x and y ,

$$\sin^2 x + \cos^2 x = 1$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \quad \text{and}$$

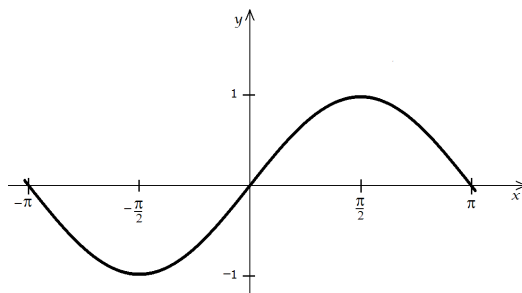
$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

Use the statements above to prove each of the following identities.

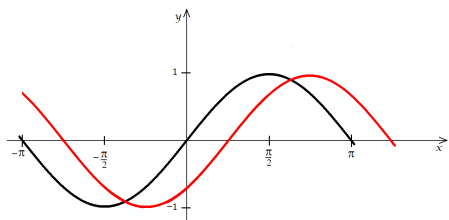
a) $\sin 2x = 2 \sin x \cos x$ b) $\cos 2x = \cos^2 x - \sin^2 x$ c) $\cos 2x = 2 \cos^2 x - 1$

d) $\cos 2x = 1 - 2 \sin^2 x$ e) $\left| \cos \frac{x}{2} \right| = \sqrt{\frac{1 + \cos x}{2}}$ f) $\left| \sin \frac{x}{2} \right| = \sqrt{\frac{1 - \cos x}{2}}$

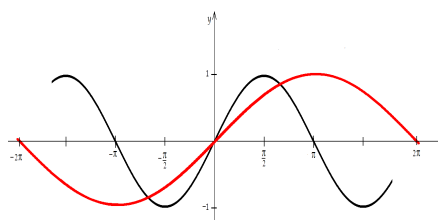
33. Given the graph of $y = \sin x$, sketch the graphs of

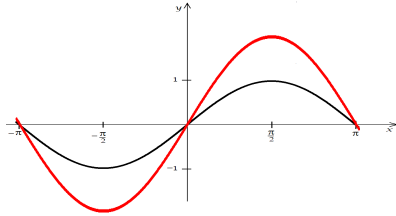
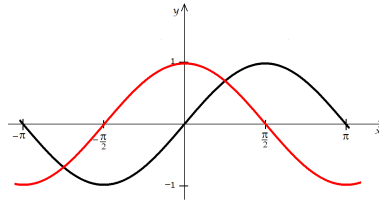
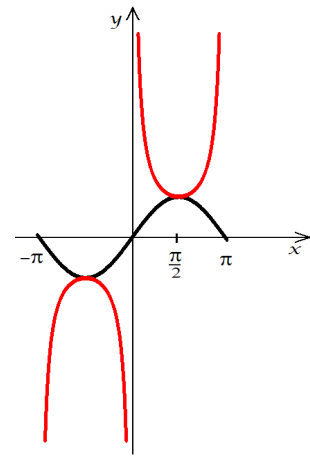


a) $y = \sin\left(x - \frac{\pi}{4}\right)$



b) $y = \sin\left(\frac{x}{2}\right)$



c) $y = 2 \sin x$ d) $y = \cos x$ e) $y = \frac{1}{\sin x}$