## Sample Problems

1. Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

2. One side of a rectangle is $7 \, \mathrm{cm}$ shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is $118 \, \mathrm{cm}$.

3. The sum of two consecutive even integers is -170. Find these numbers.

4. We have a jar of coins, all quarters and dimes. All together, they are worth $\$17.\,60$ We have 13 more quarters than dimes. How many quarters, how many dimes?

5. The sum of three consecutive odd integers is 57. Find these numbers.

6. One side of a rectangle is $3 \, \mathrm{cm}$ shorter than four times the other side. Find the sides if the perimeter of the rectangle is $204 \, \mathrm{cm}$.

7. The opposite of a number is 18 more than twice the number. Find this number.

8. Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.

9. Red pens cost $\$1$ each, blue ones cost $\$1.50$ each. We baught some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid $\$58$?

10. 55 people showed up on the party. There were 3 less women than men. How many men were there?

11. Ann and Betty dine together. The total bill is $\$38$. Ann paid $\$2$ more than Betty. How much did Betty pay?

12. A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is $\$610$. How much of each denomination of bill does he have?

13. Three times a number is one less than twice the difference of the number and three. Find this number.

14. Amy's age is three less than five times her son's age. How old are they if the sum of their ages is 33?

15. Consider the figure shown on the picture. Angles that look like right angles are right angles. Find the value of x if we know that the area of this object is 58 unit^2.

16. a) Find the value of x if the triangle shown on the picture has a perimeter 54 units.

    b) Given the value of x you found, compute the area of the triangle.

17. There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents? (Hint: think in terms of cents)

18. Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?
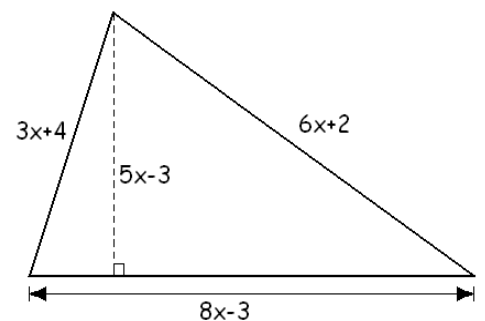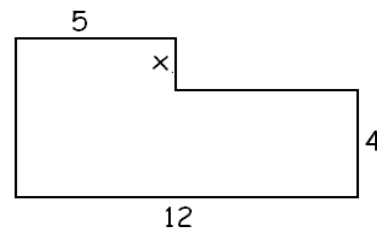
Practice Problems

1. Mary bought four less than three times the number of books that Jose did. Together they bought sixteen books. How many did Jose buy?

2. A school purchases tickets to a show. A child ticket costs $8 and an adult ticket costs $14. The school has paid a total of $610 for tickets. The number of child tickets was 5 greater than three times the number of adult ticket. How many of the tickets were for adults?

3. Julia is 5 years younger than her brother, Tom. How old are they if the sum of their ages is 43?

4. One side of a rectangle is 6 in shorter than the other side. Find the sides of the rectangle if its perimeter is 120 in.

5. One side of a rectangle is 6 in shorter than twice the other side. Find the sides of the rectangle if its perimeter is 120 in.

6. The largest angle in a triangle is three times as large as the smallest angle. The middle angle is $35°$ larger than the smallest angle. Find the angles in the triangle.

7. What a great ceremony! We had 150% more guests this year than last year. If the number of guests this year is 1875, how many guests were there last year?

8. The sum of two numbers is 27. Their difference is 11. Find these numbers.

9. The sum of two numbers is 11. Their difference is 27. Find these numbers.

10. The sum of two numbers is -11. Their difference is 27. Find these numbers.

11. The sum of two consecutive odd integers is 92. Find these numbers.

12. The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.

13. Lisa took 5 exams. The first 4 received scores of 84, 89, 64, and 82. How much did she score on the fifth exam if her average score is 80 points?

Sample Problems - Answers

1. 7 big, 17 small   2. $11\,\text{cm}$ by $48\,\text{cm}$ 3. -86 and -84 4. 41 dimes and 54 quarters

5. 17, 19, and 21   6. $21\,\text{cm}$ by $81\,\text{cm}$ 7. -6 8. 25   9. 10 blue and 43 red pens

10. 26 women and 29 men   11. Betty paid $18 and Ann paid $20

12. 56 five-dollar bills and 33 ten-dollar bills   13. -7 14. 6 and 27 years   15. $x = 2$

16. a) 3 unit   b) $84\,\text{unit}^2$ 17. 12 quarters, 58 dimes   18. 37

Practice Problems - Answers

1. 5   2. 15   3. Julia is 19 and Tom is 24   4. $27\,\text{in}$ and $33\,\text{in}$ 5. $22\,\text{in}$ and $38\,\text{in}$

6. $29°$, $64°$, $87°$ 7. 750   8. 8 and 19   9. -8 and 19   10. -19 and 8

11. 45 and 47   12. 4   13. 81

Sample Problems - Solutions

1. Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

   Solution: Let us denote the number of big ones by x. Then the number of small ones is $2x + 3$. We obtain the equation expressing the total weight:

$$
\begin{aligned}
3(2x+3) + 4x &= 79 && \text{distribute} \\
6x + 9 + 4x &= 79 && \text{combine like terms} \\
10x + 9 &= 79 && \text{subtract 9} \\
10x &= 70 && \text{divide by 10} \\
x &= 7
\end{aligned}
$$

   The number of big ones is then 7, and so the number of small ones is $2(7) + 3 = 17$. We check: the number of small ones, 17 is indeed 3 more than twice the number of big ones, 7. The total weight is $7(4) + 17(3) = 28 + 51 = 79$. Thus the solution is $\boxed{\text{7 big and 17 small}}$.

2. One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.

   Solution: Let us denote the shorter side by x. Then the longer side is $5x - 7$. We obtain the equation for the perimeter:

$$
\begin{aligned}
2x + 2(5x-7) &= 118 && \text{distribute} \\
2x + 10x - 14 &= 118 && \text{combine like terms} \\
12x - 14 &= 118 && \text{add 14} \\
12x &= 132 && \text{divide by 12} \\
x &= 11
\end{aligned}
$$

   Thus the shorter side is 11 cm, the longer side is $5(11\,\text{cm}) - 7\,\text{cm} = 48\,\text{cm}$. We check: the perimeter is $2(11\,\text{cm}) + 2(48\,\text{cm}) = 118$ cm and 48 is indeed 7 shorter than five times 11. Thus the solution is: $\boxed{11 \text{ cm by } 48 \text{ cm}}$.

3. The sum of two consecutive even integers is -170. Find these numbers.

   Solution: Let us denote the smaller number by x. Then the larger number is $x + 2$. The equation expresses the sum of the numbers.

$$
\begin{aligned}
x + x + 2 &= -170 && \text{combine like terms} \\
2x + 2 &= -170 && \text{subtract 2} \\
2x &= -172 && \text{divide by 2} \\
x &= -86
\end{aligned}
$$

   Then the larger number must be $-86 + 2 = -84$. Thus the numbers are $\boxed{-86 \text{ and } -84}$.

4. We have a jar of coins, all quarters and dimes. All together, they are worth $17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?

Solution: Let us denote the number of dimes by x. Then the number of quarters must be $x + 13$. We obtain the equation by expressing the total value, in pennies:

$$\begin{aligned}
10x + 25(x + 13) &= 1760 \quad \text{distribute} \\
10x + 25x + 325 &= 1760 \quad \text{combine like terms} \\
35x + 325 &= 1760 \quad \text{subtract 325} \\
35x &= 1435 \quad \text{divide by 35} \\
x &= 41
\end{aligned}$$

Thus we have 41 dimes and $41 + 13 = 54$ quarters. We check: $41(0.10) + 54(0.25) = 4.10 + 13.50 = 17.6$. Thus the solution is: $\boxed{41 \text{ dimes and 54 quarters}}$.

5. The sum of three consecutive odd integers is 57. Find these numbers.

Solution: Let us denote the smallest number by x. Then the other two numbers must be $x + 2$ and $x + 4$. The equation expresses the sum of the three numbers.

$$\begin{aligned}
x + x + 2 + x + 4 &= 57 \quad \text{combine like terms} \\
3x + 6 &= 57 \quad \text{subtract 6} \\
x &= 51 \quad \text{divide by 3} \\
x &= 17
\end{aligned}$$

Thus the three numbers are 17, and $17 + 2 = 19$, and $17 + 4 = 21$. We check: indeed, $17 + 19 + 21 = 57$. Thus the solution is: $\boxed{17, 19, \text{ and } 21}$.

6. One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.

Solution: Let us denote the shorter side by x. Then the longer side is $4x - 3$. We obtain the equation for the perimeter:

$$\begin{aligned}
2x + 2(4x - 3) &= 204 \quad \text{distribute} \\
2x + 8x - 6 &= 204 \quad \text{combine like terms} \\
10x - 6 &= 204 \quad \text{add 6} \\
10x &= 210 \quad \text{divide by 10} \\
x &= 21
\end{aligned}$$

Thus the shorter side is 21 cm, the longer side is $4(21) - 3 = 81$ cm. We check: the perimeter is $2(21) + 2(81) = 42 + 162 = 204$ cm and 81 is indeed 3 shorter than four times 21. Thus the solution is: $\boxed{21 \text{ cm by } 81 \text{ cm}}$.

7. The opposite of a number is 18 more than twice the number. Find this number.

Solution: Let us denote the number by x. The the two things that we are comparing are:

the opposite of the number is $-x$ and   twice the number is $2x$

Now we make these two equal by adding the difference to the SMALLER number. Since twice the number is 18 less than the opposite of the number, they will be equal once we add 18 to the smaller one.

$$\begin{aligned} -x &= 2x + 18 \quad \text{add } x \\ 0 &= 3x + 18 \quad \text{subtract } 18 \\ -18 &= 3x \qquad\quad \text{divide by } 3 \\ -6 &= x \end{aligned}$$

Thus the number is -6. Indeed, twice -6 is -12 which is 18 less than 6, the opposite of -6. Thus the number is $\boxed{-6}$.

8. Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.

Solution: let us denote the number by x. The two things we are comparing:

two times a number : $2x$

the sum of 80 and the opposite of the number   : $80 + (-x) = 80 - x$

We make these two equal by adding the difference to the smaller one:

$$\begin{aligned} 2x + 5 &= 80 - x \quad \text{add } x \\ 3x + 5 &= 80 \qquad\; \text{subtract } 5 \\ 3x &= 75 \qquad\; \text{divide by } 3 \\ x &= 25 \end{aligned}$$

Thus the number is 25. We check: twice 25 is 50 and the sum of 80 and the opposite of 25 is 55. 55 is indeed 5 more than 50. Thus the solution is: the number is $\boxed{25}$.

9. Red pens cost \$1 each, blue ones cost \$1.50 each. We baught some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$58?

Solution: Let us denote the number of blue pens by x. Then the number of red pens is $5x - 7$. The equation will express the total cost of the pens:

$$\begin{aligned} 1(5x - 7) + 1.50(x) &= 58 \quad \text{distribute} \\ 5x - 7 + 1.5x &= 58 \quad \text{combine like terms} \\ 6.5x - 7 &= 58 \quad \text{add } 7 \\ 6.5x &= 65 \quad \text{divide by } 6.5 \\ x &= 10 \end{aligned}$$

Thus we baught 10 blue and $5(10) - 7 = 43$ red pens. We check:

$$\begin{aligned} 43 &= 5(10) - 7 \\ 1(43) + 1.50(10) &= 43 + 15 = 58 \end{aligned}$$

Thus our solution is correct; we baught $\boxed{\text{10 blue and 43 red pens}}$.

10. 55 people showed up on the party. There were 3 less women than men. How many men were there?

Solution: Let us denote the number of women by x. Then $x + 3$ men showed up. The equation expresses the number of people:

$$
\begin{aligned}
x + x + 3 &= 55 \quad \text{combine like terms} \\
2x + 3 &= 55 \quad \text{subtract 3} \\
2x &= 52 \quad \text{divide by 2} \\
x &= 26
\end{aligned}
$$

Thus there were $\boxed{26 \text{ women and } 29 \text{ men}}$ on the party.

11. Ann and Betty dine together. The total bill is $38. Ann paid $2 more than Betty. How much did Betty pay?

Solution: Let us denote by x the amount that Betty paid. Then Ann paid $x + 2$. The equation expresses the total amount paid:

$$
\begin{aligned}
x + x + 2 &= 38 \quad \text{combine like terms} \\
2x + 2 &= 38 \quad \text{subtract 2} \\
2x &= 36 \quad \text{divide by 2} \\
x &= 18
\end{aligned}
$$

Thus $\boxed{\text{Betty paid \$18 and Ann paid \$20}}$.

12. A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is $610. How much of each denomination of bill does he have?

Solution: Let us denote the number of ten-dollar bills by x. Then the number of five-dollar bills is $x + 23$. We will write an equation expressing the total value of all bills.

x many ten-dollar bills are worth $10x$

$x + 23$ many five-dollar bills are worth $5(x + 23)$

So the total value is

$$
\begin{aligned}
10x + 5(x + 23) &= 610 \quad \text{distribute} \\
10x + 5x + 115 &= 610 \quad \text{combine like terms} \\
15x + 115 &= 610 \quad \text{subtract 115} \\
15x &= 495 \quad \text{divide by 15} \\
x &= 33
\end{aligned}
$$

We had x many ten-dollar bills. That is 33 ten-dollar bills. We had $x + 23$ many five-dollar bills. That is $33 + 23 = 56$ five-dollar bills. So we have $\boxed{56 \text{ five-dollar bills and } 33 \text{ ten-dollar bills}}$.

We check: the value of these bills is $5(56) + 33(10) = 280 + 330 = 610$ and 56 is indeed 23 greater than 33. So our solution is correct.

13. Three times a number is one less than twice the difference of the number and three. Find this number.

Solution: Let us denote the nuber by x. We translate the sentence to an equation and then solve for x.

$$
\begin{aligned}
3x &= 2(x-3)-1 & &\text{distribute} \\
3x &= 2x-6-1 & &\text{combine like terms} \\
3x &= 2x-7 & &\text{subtract } 2x \\
x &= -7
\end{aligned}
$$

So the number is $\boxed{-7}$. We check: three times -7 is -21 and twice the difference of -7 and 3 is $2(-7-3) = 2(-10) = -20$. Indeed, -21 is one less than -20, and so our solution is correct.

14. Amy's age is three less than five times her son's age. How old are they if the sum of their ages is 33?

Solution: Let us denote the son's age by x. Then Amy's age is $5x - 3$. The equation will express the sum of their ages:

$$
\begin{aligned}
x + 5x - 3 &= 33 & &\text{combine like terms} \\
6x - 3 &= 33 & &\text{add 3} \\
6x &= 36 & &\text{divide by 6} \\
x &= 6
\end{aligned}
$$

Thus the son's age is 6. Then Amy's age is $5 \cdot 6 - 3 = 27$. So their ages are $\boxed{6 \text{ years and } 27 \text{ years}}$. We check: 27 is indeed three less than 5 times 6. The sum of their ages is $6 + 27 = 33$, and so our solution is correct.

15. Consider the figure shown on the picture. Angles that look like right angles are right angles. Find the value of x if we know that the area of this object is 58 unit^2.
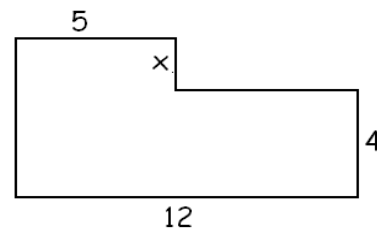
Solution: We will interpret this shape as a large rectangle from which a rectangular corner was removed.

Consider first the big rectangle. The horizontal side is 12 units long, and the vertical side is $x + 4$ units long. Therefore, the area of the big rectangle is $12(x+4)$.

Consider now the missing rectangular corner. Its horizontal side is 7 units long, its vertical side is x units long. Therefore, the area of the smaller rectangle is $7x$.

The area of the object is the difference of the areas of the two rectangles.

$$
\begin{aligned}
12(x+4) - 7x &= 58 & &\text{distribute} \\
12x + 48 - 7x &= 58 & &\text{combine like terms} \\
5x + 48 &= 58 & &\text{subtract 48} \\
5x &= 10 & &\text{divide by 5} \\
x &= 2
\end{aligned}
$$

Thus our solution is $\boxed{x = 2}$. We check: if $x = 2$, then the big rectangle is 6 unit by 12 unit with area $6 \cdot 12 = 72$ unit^2. The small rectangle is then 7 unit by 2 unit with area $7 \cdot 2 = 14$ unit^2. So the object's area is $72 - 14 = 58$ unit^2, and so our solution is correct.

16. a) Find the value of x if the triangle shown on the picture has a
perimeter 54 units.

b) Given the value of x you found, compute the area of the triangle.

Solution: The equation will express the perimeter of the triangle, the
sum of the three sides.

$$
\begin{aligned}
3x + 4 + 8x - 3 + 6x + 2 &= 54 &&\text{combine like terms}\\
17x + 3 &= 54 &&\text{subtract } 3\\
17x &= 51 &&\text{divide by } 17\\
x &= 3
\end{aligned}
$$
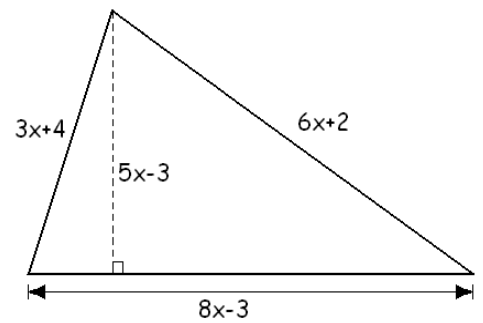
We can check by finding the sides and then computing the perimeter. $3x + 4 = 9 + 4 = 13$, $8x - 3 = 24 - 3 = 21$,
and $6x + 2 = 18 + 2 = 20$. The perimeter of the triangle is $13 + 21 + 20 = 54$ units.

b) Now that we know that x is 3, the horizontal side is $8 \cdot 3 - 3 = 21$ units, and the height is $5 \cdot 3 - 3 = 12$ units long.
Thus the area of the triangle is $\dfrac{21 \cdot 12}{2} = \boxed{84 \text{ unit}^2}$.

17. There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the
number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents?

Solution: Let us denote the number of quarters by x. Then there are $5x - 2$ dimes. The equation will express the value
of all coins, in cents.

$$
\begin{aligned}
25x + 10\,(5x - 2) &= 880 &&\text{distribute}\\
25x + 50x - 20 &= 880 &&\text{combine like terms}\\
75x - 20 &= 880 &&\text{add } 20\\
75x &= 900 &&\text{divide by } 75\\
x &= 12
\end{aligned}
$$

If $x = 12$, then $5x - 2 = 5 \cdot 12 - 2 = 58$. So there are $\boxed{12 \text{ quarters and } 58 \text{ dimes}}$. We check: 58 is indeed 2 less than
5 times 12. The total value of all coins is $0.25\,(12) + 0.10\,(58) = 3 + 5.80 = 8.80$. Thus our solution is correct.

18. Lisa took 5 exams. The first 4 received scores of $72, 93, 86,$ and 82. How much did she score on the fifth exam if her
average score is 74 points?

Solution: to take the average of five numbers, we add them, and divide the sum by 5. Let us label the missing score by
x.

$$
\begin{aligned}
\frac{72 + 93 + 86 + 82 + x}{5} &= 74 &&\text{add the numbers in numerator}\\
\frac{x + 333}{5} &= 74 &&\text{multiply by } 5\\
x + 333 &= 370 &&\text{subtract } 333\\
x &= 37
\end{aligned}
$$

So the result on the fifth exam was $\boxed{37}$. We check by computing the average of these scores.
$\dfrac{72 + 93 + 86 + 82 + 37}{5} = \dfrac{370}{5} = 74$ Thus our solution is correct.

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