1 Pre-Algebra

Parentheses do Not Mean Multiplication
The Order of Operations Agreement

1. a) In what order do we perform multiplications and divisions? Solution

2. b) In what order do we perform additioms amd subtractions? Solution

3. Perform the given operations.
   a) $(29-11)+4$
   b) $29-(11+4)$
   c) $29-11+4$ Solution

4. Perform the given operations.
   a) $40\div(5\cdot 2)$
   b) $(40\div 5)\cdot 2$
   c) $40\div 5\cdot 2$ Solution

5. Perform the given operations.
   a) 3^2+4^2
   b) $(3+4)^2$ Solution

6. Simplify each of the given expressions.
   a) $2+3\,(4+5)$
   b) $(2+3)\,(4+5)$ Solution

7. Simplify each of the given expressions.
   a) $14-4\left(3^2-7\right)$
   b) $(14-4)\left(3^2-7\right)$ Solution

8. Simplify each of the given expressions.
   a) $67-4\left(5+3\left(4\cdot 7-5^2\right)\right)$ Solution
   b) $\dfrac{24\div 4\cdot 3}{9-4+1}$ Solution
   c) $8+12\div 3+3\cdot 2^2$ Solution
   d) $2\,(7+3\,(2\cdot 8-5))$ Solution
   e) $2^4+\dfrac{41+7}{2\cdot 3}\div 4-4$ Solution
   f) $2^2-3\left(4-2\left(4\cdot 3-5^2\right)\right)$ Solution
   g) $7-4\left(5-(-4)^2+(-1)\right)+1$ Solution
   h) $\left(\left(3^2-5\right)^2-14\right)^2-1^3$ Solution

Perimeter and Area of Right Triangles

9. Simplify each of the given expressions.
   a) $48-\dfrac{6^2}{3}$
   b) $\dfrac{48-6^2}{3}$ Solution

Divisibility

10. Consider the numbers

$$159,\ 289,\ 292,\ 444,\ 591,\ 50\,020$$

Find all numbers from the list that are
   a) divisible by 3
   b) divisible by 4
   c) divisible by 12 Solution

11. List all factors of 75. Solution

12. a) Is 57 a prime number?
   b) Is 59 a prime number? Solution

13. Consider the numbers $21, 37, 77, 89, 93$. Find all prime numbers from this list. Solution

14. Is 1139 a prime number? Solution

15. Find the prime factorization of 2250. Solution

16. True or false?
   a) For any integer n, if n is divisible by 6 and by 7, then it is also divisible by 42.
   b) For any integer n, if n is divisible by 6 and by 8, then it is also divisible by 48. Solution

17. Find the greatest common factor and least common multiple for the given pair of numbers.
   a) 36 and 45 Solution
   b) 189 and 126 Solution

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18. Suppose that x is a positive integer, not 4 and not 200. Find all possible values of x so that $\gcd(4, 200, x) = 4$ and $\text{lcm}(4, 200, x) = 200$.

Solution

19. Perform the given divisions with remainder.

a) $137 \div 4$ Solution

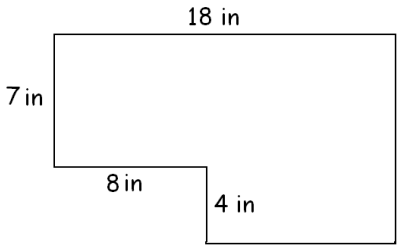
b) $325 \div 7$ Solution

20. What is the last digit of 2^{99}? Solution

21. Find the perimeter and area of the rectangle with sides 11ft and 9ft long. Solution

22. Find the perimeter and area of the right triangle with sides 10in, 24in, and 26in long. Solution

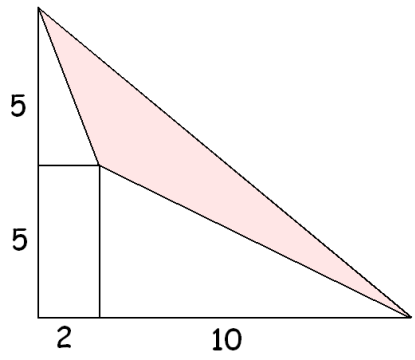
23. Find the perimeter and area of the figure shown. Solution



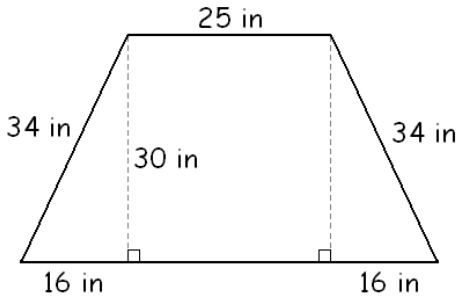
24. Suppose A is the set of all integers divisible by 5 and B is the set of all integers divisible by 10. Label each of the given statements true or false.

$A \subseteq B$ $B \subseteq A$ Solution

25. Find the area of the shaded region. Solution



26. Find the perimeter and area of the trapezoid shown on the picture. Solution

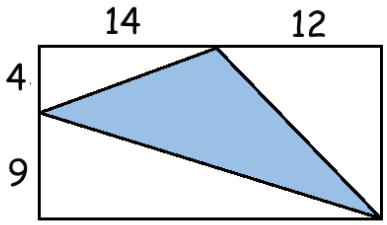


27. Plot the given points and compute the perimeter and area of the figure you got. $A(-1,-2)$, $B(1,-2)$, $C(1,3)$, and $D(-1,3)$ Solution

28. Compute the area of the parallelogram given by the points $A(-4,-1)$, $B(3,-1)$, $C(5,3)$, and $D(-2,3)$. Solution

29. Compute the area of the triangle determined by $A(-5,2)$, $B(-1,2)$, and $C-(1,2)$. Solution

30. Find the area of the shaded region. Solution



31. Simplify each of the following.

a) $5^2 - 9^2$

b) $(5-9)^2$ Solution

32. Simplfy each of the following.

a) $|8-15|$

b) $|8|-|15|$ Solution

33. Simplify each of the following.

a) $\sqrt{25}+\sqrt{144}$

b) $\sqrt{25+144}$ Solution

34. Simplify $\sqrt{29-\sqrt{16}}$ Solution

35. Simplify $7-2(-3)$ Solution

36. Simplify each of the following.

a) $11-5(4-7)$ Solution

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37. Simplify each of the following.

a) -3^3 b) $(-3)^2$ c) $-(-3)^2$

Solution

38. Simplify each of the following.

a) $4-(-5)+7$

b) $(4-(-5))+7$

c) $4-((-5)+7)$ Solution

39. Simplify each of the given expressions.

a) $-5-7(-4-6)$

b) $-(5-7(-4-6))$

c) $-(5-7(-4))-6$

d) $(-5-7)(-4-6)$

e) $-(5-7(-4)-6)$

f) $-(5-7-(4-6))$

g) $-5-7-(4-6)$

h) $-(5-7)(-4-6)$

i) $-(5-7)-4(-6)$

j) $-5(-7)-4(-6)$ Solution

40. Simplify each of the given expressions.

a) $-((-3)+(5-1))-7^2$ Solution

b) $\dfrac{5^2-3^2}{14-3\cdot 2^2-2}$ Solution

b) $-2(-1+10(7-5))-(-7)^2$ Solution

c) $\sqrt{(2-(-4))(9-3\div 3)-(-1)^3}$ Solution

41. Simplify each of the following.

a) $|-3-8|-|11+9|$

b) $|-3-8-|11+9||$

c) $|-3-8|-11+9||$

d) $|-3-|8-11+9||$

e) $|-3|-8-11+9||$ Solution

42. Simplify each of the following.

a) $|-12-7|-|1+11|$

b) $|-12-7|-1+11||$

c) $|-12|-7-1+11||$ Solution

43. Evaluate each of the following.

a) $3a^2-2b+4$ if $a=2$ and $b=5$

b) $3a^2-2b+4$ if $a=5$ and $b=2$

c) $3x^2-5x+8$ if $x=3$ Solution

44. Evaluate each of the following.

a) $\dfrac{n-4}{k}$ if $k=-1$ and $n=7$. Solution

b) $-x^2+3x-5$ if $x=-8$ Solution

c) $11a+4b-2c$ if $a=5$, $b=3$, and $c=-2$
Solution

d) $\dfrac{9-(6a+b)}{ab+10}$ if $a=-2$ and $b=5$ Solution

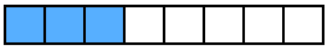
e) $2x^2-3y^3$ if $x=5$ and $y=-2$ Solution

45. Given the values -4,-2, and 6, find all numbers listed that are solution of the equation
$x^3-x^2-27x+2=x^2-3x+2$. Solution

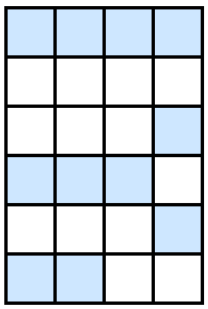
46. Is the statement $9x^2+9x+3<2x+104$ true or false when $x=3$? Solution

47. Express the shaded area as a fraction.

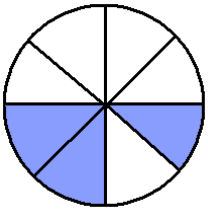
a) Solution



b) Solution



c) Solution



48. Find each of the following.

a) $\dfrac{1}{6}$ of 18

b) $\dfrac{5}{6}$ of 18 Solution

49. Find 9% of 200. Solution

50. In a class, there are 29 students. 9 of the students are men. What fraction of the students are men?
Solution

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51. There are 800 students enrolled in a school. If $\frac{3}{4}$ of the students are boys, how many students are girls? Solution

52. A \$500 TV went on a 6% off sale. What is the sale price? Solution

53. a) Rewrite 48% as a reduced fraction. Solution

    b) Re-write $\frac{4}{5}$ with a denominator 10. Solution

54. In my left pocket, I have 11 dollars and 87 cents. In my right pocket, I have 2 dollars and 65 cents. How much money do I have all together? Solution

55. Bring $\frac{5}{11}$ and $\frac{4}{9}$ to to a common denominator in order to compare the two fractions. Use the least common denominator. Solution

56. Re-write as percent:

    a) $\frac{19}{25}$ b) $\frac{3}{5}$ Solution

57. a) If the price went up from \$1200 to \$1560, what percentage of a change is this?

    b) 1560 is what percent of 1200? Solution

58. a) 80% of a number is 72. Find this number. Solution

    b) 125% of a number is 450. Find this number. Solution

59. We decreased quantity Q by 5%. Which of the given expressions does that?

    $0.5Q$ $0.05Q$ $95Q$ $0.95Q$ $1.05Q$ Solution

60. Which of the following expresses an 80% increaseof quantity Q.

    A) $9Q$ C) $1.08Q$ E) $0.8Q$
    B) $80Q$ D) $1.8Q$ F) $180Q$
    Solution - Youtube link

61. Express each of the given expressions as a percentage change from quantity Q. Solution

    $1.8Q$ $1.08Q$ $0.92Q$
    $8Q$ $9Q$ $0.2Q$

62. 125% of a number is 450. Find this number. Solution

63. a) We increased \$3000 by first 20% and then later by another 30%. What is the resulting number? Express the two changes as a single change. What percentage is this change? Solution

    b) We increased a quanttity by 25% and then later decreased it by 8%. Express the two changes as a single change. What percentage is this change? Solution

64. a) A TV went on a 15% off sale. The sale price is \$238. What was the original price? Solution

    b) Jake got an 8% raise. Now he is making \$3510 a month. How much was he making before the raise? Solution

65. Simplify each of the following.

    a) $\left(\frac{1}{5}-\frac{1}{7}\right)\div\frac{24}{35}$ Solution

    b) $-\frac{9}{10}-\left(-\frac{3}{4}\right)$ Solution

    c) $\left(-\frac{1}{2}\right)^2-\frac{3}{10}$ Solution

    d) $2-\frac{3}{4}\cdot\frac{1}{6}$ Solution

66. Evaluate $-x^2-2x+3$ if $x=-\frac{3}{5}$
    Solution -Youtube link

67. Verify that the repeating decimal $1.174747474..... = 1.1\overline{74}$ represents a rational number by converting it into a fraction of two integers. Solution

68. Solve each of the given equations.

    a) $\frac{x+12}{5}=6$ Solution

    b) $\frac{x-4}{3}=5$ Solution

69. Solve each of the given equations.

    a) $3x+8=-7$ Solution

    b) $-5x+3=38$ Solution

    c) $\frac{x+20}{3}=2$ Solution

    d) $\frac{x+3}{-5}=2$ Solution

    e) $\frac{x-2}{5}=-1$ Solution

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f) $\dfrac{x}{-4}+5=8$ Solution

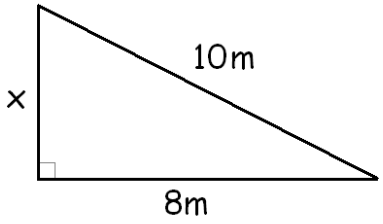
70. Solve each of the given equations.
a) $-\dfrac{2}{3}x-\dfrac{1}{8}=\dfrac{13}{24}$ Solution

71. Ben took five exams. Four of the exams were 71, 75, 67 and 93 points. What was the score on the fifth exam if the average of all five exams was 78 points.
Solution - Youtube link

Weighted Averages

72. Prove that the repeating decimal $0.78257257257...$ is rational by re-writing it as a quotient of two integers. You do not need to reduce the fraction to lowest terms.
Solution

73. Find the missing side in the right triangle shown on the picture if we know that the area of the triangle is 24m^2. Solution



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2 Introductory Algebra

The Words AND and OR

1. Determine whether the statements given are true or false.

   a) $3 \leq 3$ b) $5 \leq 3$ c) $2 \leq 10$ Solution

2. Determine whether the given statements are true or false.

   a) 34 is not divisible by 7, or 39 is divisible by 9.

   b) 34 is not divisible by 7, and 39 is divisible by 9. Solution

3. Determine whether the given statements are true or false.

   a) Seven is less than nine, and 5 is odd.

   b) Six is a natural number and zero is a natural number.

   c) Four is less than five, and five is even. Solution

4. Determine whether the given statement is true or false. *The empty set is a subset of any set, or the number* 11 *is even.* Solution

5. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers x from this list such that

   a) $x \geq 6$ and x is odd

   b) $x \geq 6$ or x is odd Solution

6. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers x from this list such that

   a) $x < 7$ or $x > 3$

   b) $x < 7$ and $x > 3$ Solution

7. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers x from this list such that

   a) $x < 7$ and $x < 4$

   b) $x < 7$ or $x < 4$ Solution

Introduction to Set Theory

8. Suppose that $A = \{1, 5, 6, 8, 10\}$ and $B = \{5, 6, 10\}$. Determine whether the given statements are true or false.

   a) $4 \in A$ b) $B \subseteq A$ Solution

9. Which statement is true?

   $\{1,\ 3,\ 6\} \subseteq \{1,\ 2,\ 3,\ 4\}$

   $\{1,\ 3,\ 6\} \not\subseteq \{1,\ 2,\ 3,\ 4\}$ Solution

10. Which statement(s) are true?

    $\{b, c, d, e\} \subseteq \{a, b, c, d, e, f, g, h\}$

    $\{b, c, d, e\} \not\subseteq \{a, b, c, d, e, f, g, h\}$ Solution

11. Which statement is true?

    $\{2,\ 4,\ 6\} \subseteq \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

    $\{2,\ 4,\ 6\} \not\subseteq \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$ Solution

12. Which statement is true?

    $\{-1, 0, 2\} \subseteq \mathbb{N}$

    $\{-1, 0, 2\} \not\subseteq \mathbb{N}$ Solution

13. True or false?

    $\{x \mid x \text{ is a cat}\} \subseteq \{x \mid x \text{ is a black cat}\}$

    $\{x \mid x \text{ is a cat}\} \not\subseteq \{x \mid x \text{ is a black cat}\}$ Solution

14. Suppose that E is the set of all even numbers and O is the set of all odd numbers. Which of the given statements are true?

    $E \subseteq O$ $O \subseteq E$ Solution

15. Two sides of a triangle are 5 units and 13 units long.

    a) What lengths are possible for the third side? Solution

    b) What lengths are possible for the third side if it is to be the longest side? Solution

    c) What lengths are possible for the third side if it is to be the shortest side? Solution

16. Three sides of a triangle are different integers. What is the smallest possible value of the perimeter of this triangle? Solution

17. Simplify each of the following.

    a) $(2x + 11y + 3) + (20x - 11y + 9)$ Solution

    b) $4(4x - 3) - 3(-x + 2)$ Solution

    c) $-5(3a + 4(7a + 3(-3a - 4)))$ Solution

    d) $-2(6x - 3(3x - 2(2x - 5)))$ Solution

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e) $(2x+1)^2+(2x-1)^2$ Solution

f) $(3x+5)^2-(3x-5)^2$ Solution

g) $(2x-1)^2-(3x+2)(4x-3)$ Solution

h) $\left(3x^3-2\right)^2$ Solution

i) $\left(2x^4-5y^3\right)^2$ Solution

18. a) Jake studies x hours for the test. Julie studied twice as long. Express the time, in terms of x, that Julie spent studying. Solution

b) Ann spent 18 dollars less than Juan. If Juan's spending is denote by x, express Ann's spending in terms of x.Solution

c) We purchased y many posters, each for \$4. Write an algebraic expression for the amount of money we spent. Solution

d) A roof's length is 7 feet less than twice its width. If we let t represent the width, write an algebraic expression for the length. Solution

19. Translate the given expression.

The sum of a number increased by one and the number decreased by six. Solution

20. Voltage taken by resistor 1 is ten more than four times the voltage taken by resistor 2. Express the voltage taken by resistor 1 in terms of the voltage taken by resistor 2. Use x for the variable. Solution

21. Translate the given sentence to an equation using x for the number.

a) The sum of three times a number and six is nine. Solution

b) The sum of a number and four is ten. Solution

c) The product of three more than a number and eight is 48. Solution

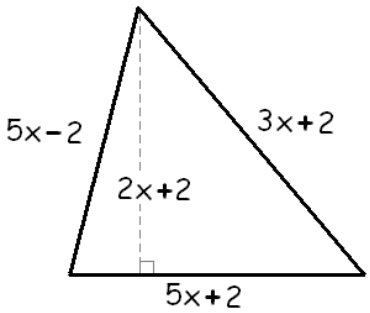
22. Translate the given sentence to an equation using x for the number. Then solve the equation for x.

a) The sum of a number and four is ten. Find this number. Solution

b) The sum of three times a number and six is nine. Find this number. Solution

c) The product of three more than a number and eight is 48. Find this number. Solution

23. Express the perimeter of the triangle in terms of x. Solution



24. Solve each of the given equations.

b) $\dfrac{\dfrac{x-5}{-4}+5}{4}-1=4$ Solution

c) $4(3x+1)-(x+3)=10x-4$ Solution

d) $\dfrac{x+5}{7}-\dfrac{2x+1}{9}=x+6$ Solution

e) $\dfrac{1}{10}(x-14)=\dfrac{5}{2}x-\dfrac{3}{5}$ Solution

f) $-3(x+4(x+3(-3x-1)))=-3+7(8x-5)$ Solution

g) $-5(x+2(x-2(-4x+3)))=60(-5x+1)$ Solution

h) $(x+2)(2x-3)=2(x+1)^2-17$ Solution

25. Solve each of the given inequalities.

a) $\dfrac{3x+6}{-4}\geq 1$ Solution

b) $4(3x-7)-5(2x-5)>-(-3x+2)+1$ Solution

c) $\dfrac{x-4}{5}-\dfrac{5x-1}{2}>-2x-3$ Solution

26. There were 7 more women than men at the party. If there were 31 guests, how many of them were women? Solution

27. Susan is asked about her age. She answers as follows. "*My age is 8 years less than twice the age of my brother.*" How old is her brother if Susan is 24 years old? Solution

28. The cost of a taxi ride is \$1.55 for the first mile and \$1.35 for each additional mile or part thereof. Find the distance we covered if we paid \$24.50. Solution

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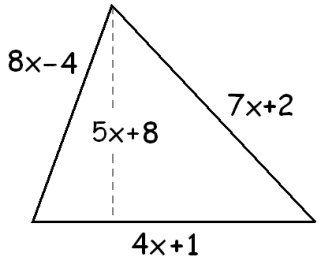
29. Wendy is asked about her age. She answers as follows. "My age is seven years less than twice the age of my son. The sum of our ages is 61 years. How old is Wendy?" Solution

30. Find the value of x if the triangle shown on the picture has a perimeter of 189 uits. Solution



31. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost 11 dollars, adult tickets cost 14 dollars. The number of children tickets purchased was five less than twice the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of 465 dollars? Solution

32. The area of a square would increase by 17 square-yards if we increased its sides by 1 yard.. How long is a side of the square now (before the increase)? Solution

33. Five times the sum of four and a number is six less than seven times the same number. Solution

34. Suppose that $U = \{1,2,3,...,10\}$, $A = \{2,6,9,10\}$, and $B = \{1,2,3,7,10\}$. Find $A \cap B$. Solution

35. Suppose that $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{x \in U : x \leq 2\}$, and $B = \{x \in U : x \geq 7\}$. Find each of the following sets.

a) $A \cup B$ b) $A \cap B$ Solution

36. Suppose that $U = \{1,2,3...,10\}$, $A = \{1,3,5,7\}$, and $B = \{1,2,3,4\}$. Compute $B\cap$

37. Suppose that $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{x \in U : x < 8\}$, and $B = \{x \in U : x > 4\}$. Find each of the following sets.

a) $A \cup B$ b) $A \cap B$ Solution

38. Suppose that $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{2,4,6,8,9\}$, and $B = \{1,2,5,6,9\}$. Find each of the folllowing.

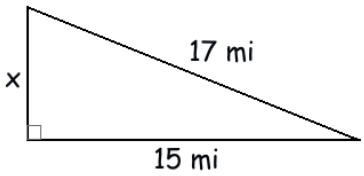
a) $\overline{A} \cap B$ b) $\overline{A} \cup \overline{B}$ c) $\overline{A \cup B}$ Solution

39. Suppose that A and B are sets such that $A \cap B = \{1,2,5\}$ and $A \cup B = \{1,2,3,4,5,6\}$. How many sets are possible for A? Solution

40. State the coordnates of the points plotted. Solution

41. Compute the area of the triangle determined by the points $A(-5,2)$, $B(-1,2)$, and $C(-1,-3)$. Solution

42. Find the exact value of x based on the picture. Solution



43. Solve each of the given systems of equations.

a) Solution

$$\begin{aligned} x + 4y &= -22 \\ y &= 3x - 12 \end{aligned}$$

b) Solution

$$\begin{aligned} (x-2)^2 + (y+3)^2 &= x^2 - y(-1-y) - 42 \\ (x+1)(y-1) &= xy - 14 \end{aligned}$$

44. Find the exact value of the distance between the points $A(-6,4)$ and $B(6,-1)$. Solution

45. Graph each of the following.

a) $y = \frac{2}{3}x + 1$ Solution

b) $4x + 3y = -12$ Solution

c) $3x - 4y = -8$ Solution

d) $2x + 3y = -5$ Solution

46. Compute each of the given sums.

a) $1 + 2 + 3 + ... + 100$ Solution

b) $3 + 6 + 9 + ... + 2019$ Solution

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c) $30+37+44+...+2018$ Solution

d) $\frac{1}{1001}+\frac{2}{1001}+\frac{3}{1001}+...+\frac{1000}{1001}$ Solution

47. The first row in a theater has 31 seats in it. The second row has one more seats than the first row. The third row has one more seats than the second row. And so on, each row has one more seats than the row before. If the last row has 191 seats in it, how many seats are there in the entire theater? Solution

48. There is a farm where chickens and cows live. All together, there are 66 heads and 202 legs. How many chickens, how many cows? Solution

49. We have 260 coins, all nickels and dimes, in the value of 20.25 dollars. How many of each? Solution

50. Find the perimeter and area of the parallelogram determined by the points $A(-2,-3)$, $B(21,-3)$, $C(5,9)$, and $D(-18,9)$. Solution

51. There are 85 men and 15 women in a large conference room.

a) How many handshakes would take place if all person shook hands with all other people in the room?

b) How many handshakes would take place if all men shook hands with all other men in the room, and all women shook hands with all women in the room, but no man shook hands with any woman?

c) How many handshakes would take place if all men shook hands with all woman, but no two men or two women shook hands? Solution

52. Simplify each of the given expressions.

a) $(3x+5)+(-4x+1)$

b) $(3x+5)-(-4x+1)$

c) $-2(3x+5)-3(-4x+1)$

d) $(3x+5)(-4x+1)$ Solution

53. Simplify each of the given expressions.

a) $-(5x-1)^2$ Solution

b) $\left(2z^5+1\right)\left(2z^5-1\right)$ Solution

54. Simplify each of the given expressions.

a) $\left(-3x^4\right)\left(-2x^5\right)$ Solution

b) $\frac{\left(x^4\right)^6}{x^4x^6}$ Solution

c) $\left(\frac{-12\,x^{10}\,y^4\,z^5}{15\,x^6\,y^4\,z^3}\right)^3$ Solution

d) $\frac{\left(x^2y^5\right)^3\left(-x^4y^3\right)^6}{\left(-xy^4x^4\right)^5}$ Solution

e) $\frac{-18\,(-x)^{10}\left(y^2\right)^8}{24x^2\,(-y)^5\,x^5}$ Solution

55. Solve each of the following.

a) $4(x+1)(x-2)=0$

b) $x(x+1)(x-2)=0$ Solution

56. Solve each of the following.

a) $(x+7)(x-8)=0$ Solution

b) $-2x(3x+1)=0$ Solution

c) $-x^6=3x^5$ Solution

d) $6x^7=3x^6$ Solution

e) $x^6=16x^4$ Solution

f) $x^2-2x=0$ Solution

57. Convert $50\,202\,000$ to scientific notation. Solution

58. Perform the given multiplication. Present your answer in scientific notation.

$\left(1.2\cdot10^5\right)\left(8.5\cdot10^7\right)$ Solution

59. If we increase one side of a square by 1 unit and decrease the other side by 5 units, the resulting rectangle's area is 77 square-unit less than the area of the original square. How long was the side of the original square? Solution

60. Factor out the GCF (greatest common factor) in $35x^7+10x^6+55x^4$. Solution

61. Factor out -1 in the given expression $-6x^5+x^3-5$ Solution

62. completely factor each of the following.

a) $12x+75$

b) $12x^2+75$

c) $12x^2-75$ Solution

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63. Completely factor each of the following.
    a) $3y^5 - 12y^3$ Solution
    b) $-2m^4 + 32$. Solution
    c) $-98x^4 + 50$ Solution
    d) $\left(2x^2 + 3x - 4\right)^2 - \left(2x^2 - 3x + 5\right)^2$ Solution

64. Solve each of the given equations.
    a) $x^2 - 2x = 0$ Solution
    b) $x^6 = 16x^4$ Solution
    c) $6x^7 = 3x^6$ Solution

65. Simplify wach of the given expressions.
    a) $\dfrac{2 - 7x}{7x - 2}$ Solution

66. If we raise a number x to the third power, the result is the same as 36 times the number. Find all possible values of x. Solution

67. Find all numbers whose square is seven greater than six times the number. Solution

68. Find the prime factorization of 12^{50}. Solution

69. Suppose that $A = 3^{500}$. Express each of the following in terms of A.
    a) 3^{501}
    b) $6 \cdot 3^{500} - 3^{502}$
    c) 3^{499}
    d) 9^{500}
    e) 3^{1500} Solution

70. Simplify the given expression. $\dfrac{4^{63} - 4^{60}}{4^{60}}$. Solution

71. Simplify each of the given expressions. Present your answer using only positive exponents.
    a) $\dfrac{3}{x^{-5}}$
    b) $\left(\dfrac{24}{8x}\right)^{-2}$
    c) $\dfrac{x^7}{3x^{-4}}$
    d) $\dfrac{x^{-4}}{x^{-7}}$ Solution

72. Simplify each of the given expressions. Present your answer using only positive exponents.
    a) $\dfrac{x^{-7}y^{-2}}{x^{-4}y^{-6}}$ Solution
    b) $\dfrac{\left(x^{-4}\right)^5}{\left(x^3\right)^{-8}}$ Solution
    c) $\dfrac{12x^{-7}\left(y^2\right)^{-5}}{15xy^{-3}x^2}$ Solution
    d) $\dfrac{-10\left(-a\right)^6\left(b^{-1}\right)^3}{8a^{-3}\left(-b\right)^7 a^5\left(-b\right)^{-4}}$ Solution
    e) $\dfrac{\left(x^6y^{-2}\right)^3\left(-x^5y^3\right)^{-2}}{\left(-x^2y^2x\right)^{-3}}$ Solution
    f) $\dfrac{3^{-1} + 5^{-1}}{3^{-1} - 5^{-1}}$ Solution
    g) $6^{-1} - 5 \cdot 2^{-3}$ Solution

73. Rewrite the given set using interval notation.Solution

$$\{x : x \leq -7 \text{ or } x > -1\}$$

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3 Intermediate Algebra

Completing the Square Part 0, Part 1

Smallest Value of a Quadratic Expression - 1

1. Factor each of the following completely by completing the square.

   a) $-3a^2 + 12a + 15$ Solution

   b) $-x^2 + 2sx - 1$ Solution

   c) $-2x^2 + 12x - 50$ Solution

2. Solve each of the given equations by completing the square.

   a) $-3x^2 + 24x - 36 = 0$ Solution

   b) $3x^2 - 6x + 3 = 0$ Solution

   c) $-2x^2 + 8x - 10 = 0$ Solution

   d) $(x-8)(x-6) = 24$ Solution

   e) $3x^2 + 30x = -66$ Solution

   f) $15x^2 + x - 2 = 0$ Solution

3. Factor by completing the square.

   a) $x^2 - 3x - 10$ Solution

4. Find the smallest value of each of the following expressions.

   a) $x^2 - 10x + 14$

   b) $(2a-1)^2 + 8$

   c) $x^2 - 8x - 180$ Solution

5. Graph $y = x^2 - 4x - 5$, State the coordinates of at least five points, including vertex and intercepts. Solution

6. Simplify each of the given expressions.

   a) $\dfrac{x^3 - 16x}{x^2 + 6x + 8}$ Solution

   b) $\dfrac{x^2 + 8x + 7}{x^3 + 6x^2 + 5x}$ Solution

7. Match the expressions with the descriptions.

   i) $(x+6)^2$ iii) $(x+6)^2 + 25$

   ii) $(x+6)^2 - 4$ iv) $(x+6)^2 - 3$

   Solution

   1) The expression .can be factored over the real numbers but not over the integers.

   2) The expression .can be factored over the integers into two different linear factors.

   3) The expression can not be factored over the integers or over the real numbers.

   4) The expression is factored over the integers into two identical factors.

8. One side of a rectangle is 12 inches shorter than three times another side. Find the sides of the rectangle if its area is 1575 square-inches. Solution

9. Find all numbers whose square is seven greater than six times the number. Solution

10. We are standing on the top of a 720ft tall building and throw a small object upward. The object's distance from the ground, measured in feet, after t seconds is $-16t^2 + 128t + 1344$. How long until the object hits the ground? Solution

11. Mia was asked about her age. Her answer was: my sister's age is ten years less than three times my age. The product of our ages is 312. How old are the sisters? Solution

12. We are organizing a fundraising event. We find that if we set the price at \$40, then we can sell 350 tickets. For each dollar increase in the price, five less tickets will be sold.

    a) How much money will we raise from the ticket sales if we set the price of a ticket to be \$50?

    b) What price(s) will result in a total income of \$14 625? Solution

13. Simplify each of the following.

    a) $\sqrt{100}$ e) $\sqrt[3]{8}$ Solution

    b) $-\sqrt{100}$ f) $-\sqrt[3]{8}$

    c) $\sqrt{-100}$ g) $\sqrt[3]{-8}$

    d) $-\sqrt{-100}$ h) $-\sqrt[3]{-8}$

14. Simplify each of the following.

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a) $\left(\sqrt[5]{-3}\right)^{20}$

b) $\left(\sqrt[4]{-3}\right)^{20}$ Solution

c) $\sqrt{x^{16}}$ Solution

15. Simplify each of the given expressions.

a) $\sqrt{90}-3\sqrt{40}+\sqrt{10}$ Solution

b) $\left(5\sqrt{2}-1\right)\left(3\sqrt{2}+5\right)$ Solution

c) $\left(4\sqrt{2}-5\right)^2$ Solution

d) $\left(3-\sqrt{10}\right)^2-\left(3-\sqrt{10}\right)\left(3+\sqrt{10}\right)$ Solution

16. a) Rationalize the denominator in $\dfrac{6}{2-\sqrt{7}}$ Solution

b) Rationalize the denominator in $\dfrac{\sqrt{5}+3}{\sqrt{5}-1}$ Solution

17. Re-write $\sqrt[3]{x^7}$ using rational exponents. Solution

18. Simplify each of teh following.

a) $\dfrac{x^{1/7}}{x^{1/14}}$

b) $\dfrac{x^{3/5}}{x^{2/3}}$

c) $\left(x^{5/8}\right)^{2/15}$

d) $\left(\dfrac{x^{7/5}}{x^{3/5}}\right)^{5/6}$ Solution

19. Simplify each of the following.

a) $\left(\dfrac{a^{3/5}}{b^{1/3}}\right)^3\left(\dfrac{b^{1/2}}{a^{5/6}}\right)$ Solution

20. Use exponential notation to simplify

a) $\sqrt[20]{x^{12}}$. Solution

21. Find the wxact value of $-2x^2-x+1$ if $x=3-2\sqrt{5}$ Solution

22. Solve the given compound inequalities.

a) $-x+2\le 6$ or $-4x+1<-19$

b) $-x+2\le 6$ and $-4x+1<-19$ Solution

23. One number is ten greater than another. What is the smallest possible value oft their product? Solution

24. a) How many four-digit numbers can be formed using only the digits 1,3, 4, 5, and 7?

b) How many four-digit numbers can be formed using only the digits 1,3, 4, 5, and 7 if repetition of digits is not allowed? (i.e 1513 not allowed) Solution

25. Suppose that $D=\{1,2,3\}$, and $Y=\{a,b,c,d,e\}$.

a) How many functions are possible with domain D and range R, where R is a non-empty subset of Y?

a) How many one-to-one functions are possible with domain D and range R, where R is a non-empty subset of Y?

a) How many relations are possible with domain D and range R, where R is a non-empty subset of Y? Solution

26. Consider the triangle with sides 8, 16, and 17 units long. Is it a right triangle? Solution

27. Find the exact value of the distance between the points $A\,(3,-6)$ and $B\,(-3,1)$. Solution

28. Two sides of a right triangle are 9 and 40 units long. Find the exact value of the third side. Solution

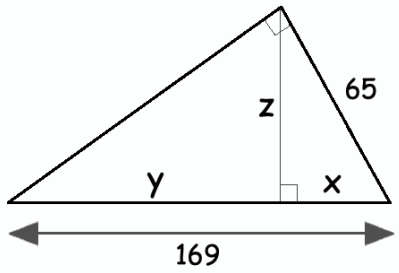
29. The base of a straight pyramid is a square with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. Solution

30. The hypotenuse of a right triangle is 34cm long. The difference between the other two sides is 14cm. Solution

31. The shortest side of a right triangle is 24 miles long. The difference between the other two sides is 4 miles. Find the missing sides. Solution

32. The base of a straight pyramid is a ssquare with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. Solution

33. Find the exact values of x, y, and z. Solution



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4 General Education Mathematics

Geometry 1
Part 1.: Standard labeling, Sides and Angles
Part 2: The sum of the Inner Angles of triangles.
Part 3. Angles of Polygons

Introduction to Set Theory
More Set Operations
Introduction to Combinatorics
Fundamental Counting Principle and Permutations
Listing and Counting Subsets
More Listing
Introduction to Probabilities
Combinations

1. Suppose that $U = \{1, 2, 3, ..., 12\}$, $A = \{1, 2, 3, 7, 8\}$, $B = \{3, 4, 5, 7, 8, 12\}$, and $C = \{2, 3, 4, 6, 7, 11, 12\}$. Find each of the given sets.

   a) $(A \cup B) \setminus C$

   b) $A \cup (B \setminus C)$ Solution

2. 12 women and 8 men will compete in a race. What is the probability that the first three finishers are

   a) all mem

   b) 2 men and 1 women? Solution

3. We toss a coin ten times. What is the probability that the results will be

   a) exactly 3 heads

   b) at least 3 heads? Solution

4. There are 7 blue and 3 red marbles in a bag. We randomly pull two marbles, with replacement. Find each of the probabilities.

   a) Both marbles pulled are blue..

   b) Both marbles pulled are red.

   d) The marbles we pulled have the same color.

   e) The marbles we pulled are of different colors.

   f) g) h) i) Find the same probabilities if there is no replacement. Solution

5. There are 18 marbles in a bag: 6 red, 4 green, and 8 blue. We randomly pull three marbles. Find each of the following probabilities with replacement:

   a) We pull 3 red marbles.

   b) We pull 3 marbles of the same color.

   c) We pull three marbles, all different colors.

   d) e) f) Find the same probabilities if there is no replacement. Solution

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6. There are 16 red and 4 blue marbles in a bag. We randomly pull two marbles. If both are red, we win \$1. If both blue, we win \$4. Otherwise, we lose \$2. What is the expected value of this game for us, with and without replacement? Solution

7. We roll a fair die. If we roll x and x is odd, we win the square of x in dollars. If x is even, we win the square of x in dollars. What is the expected value of this game for us? Solution

8. There are 12 marbles in a bag. We randomly pull a marble. If it is red, we win \$8. If it is blue, we lose \$7. How many redmarbles are i the bag if the expected value of this game for us is -0.75 dollars? Solution

9. We roll two dice. If the numbers rolled are different, we pay 2 dollars. If we roll the same number, then we will win the sum of the number rolled (i.e if we roll a 5 and a 5, we win \$10). What is the expected value of this game for us? Solution

10. Find the present value of the following three payments: \$1400 today, \$1400 a year from today, and \$1400 two years from today. Assume an annual compound interest rate of 6%, compounded monthly. Solution

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5 College Algebra

1. Solve $\dfrac{5x-4}{3x-2} < \dfrac{7}{4}$

   a) Solution 1

   b) Solution 2

2. Find an equation for the tangent line drawn to the circle $(x-4)^2 + (y-9)^2 = 40$ to the point $P(10,11)$.

   Solution

3. Find the domain of each of the given functions.

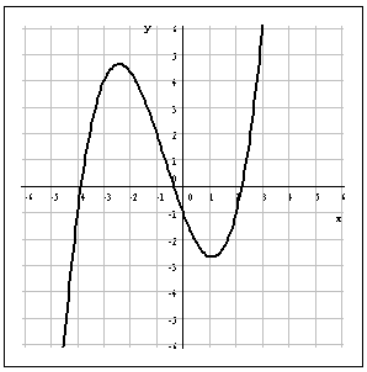
   a) $f(x) = \dfrac{1}{x^2-2x-35}$

   b) $f(x) = \sqrt{x^2-2x-35}$

   c) $f(x) = \ln\left(x^2-2x-35\right)$ Solution

4. Find an equation for the set of all points that are twice as far from $A(-3,-1)$ than from $B(9,5)$. Solution

5. Given the graph of $y = f(x)$. Graph $g(x) = |f(x)|$ in the same coordinate system. Solution



Graphing Factored Polynomials

   a) Part 1A

   b) Part 1B

   c) Part 1C

Graphing Faxtored Polynomials - Part 2

Logarithms 2

   Part 0 - the Ingredients
   Part 1 - Rule 3
   Part 2 - Rule 4
   Part 3 - Rules 5 and 6
   Part 4 - Final Thoughts

6. Suppose that $A = \log_3 2$. Express each of the following in terms of A.

   a) $\log_3 18$

   b) $\log_3 24$

   c) $\log_3\left(\dfrac{27}{4}\right)$

   d) $\log_2 3$

   e) $\log_{18} 24$ Solution

Why i^2 is -1?

Summation Notation

7. Compute each of the given sums. (Note: the solution uses $1+2+...+n = \dfrac{n(n+1)}{2}$ after induction, and there is no mentioning of arithmetic sequences.)

   a) $\sum_{k=1}^{50}(8k-3)$

   b) $\sum_{k=0}^{50}(8k-3)$

   c) $\sum_{k=20}^{50}(8k-3)$

   Solution

8. Compute the given sums.

   a) $3^2+6^2+9^2+...+120^2$ Solution

   b) $0.8^2+1.6^2+2.4^2+...+120^2$ Solution

   c) $150^2+151^2+.....+200^2$ Solution

   d) $1^2+3^2+5^2+...+201^2$ Solution

   e) $\sum_{k=5}^{200}\left(k^2-2k+1\right)$ Solution

9. Find the slope of the tsngent line drawn to the graph of $y = -\dfrac{1}{2}x^2 + 3x + 1$ from the point $P(4,7)$. Solution

10. Find all values of p so that $y = 4x+2$ is tangent to $y = px^2+8$. Solution

Solving Quadratic Inequalities

How to take the Absolute Value of a Graph

Discontinuities of Rational Functions
hole or vertical asymptote?

The Binomial Theorem

Why 0!=1

Pascal's Triangle

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6 Trigonometry

1. Consider the triangle with sides 8, 16, and 17 units long. Is it a right triangle? Solution

2. Find the exact value of the distance between the points $A\,(3,-6)$ and $B\,(-3,1)$. Solution

3. Two sides of a right triangle are 9 and 40 units long. Find the exact value of the third side. Solution

4. Find the exact value of the height of the regular triangle with sides 14 units long. Solution

5. The base of a straight pyramid is a square with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. Solution

6. The hypotenuse of a right triangle is 34cm long. The difference between the other two sides is 14cm. Solution

7. The shortest side of a right triangle is 24 miles long. The difference between the other two sides is 4 miles. Find the missing sides. Solution

8. Find the perimeter and area of the parallelogram determined by the points $A\,(-2,-3)$, $B\,(21,-3)$, $C\,(5,9)$, and $D\,(-18,9)$. Solution

9. Solve the given equations.

   a) $\sin\alpha + 1 = 2\cos^2\alpha$ Solution

Proving the Sum Formulas Using Vectors

Deriving All Other Compound Angle Formulas

Sum-Product Identities

10. Suppose that C is the center of a circle with radius 9in and a point P is at a distance of 14in from C. Find exact value of sine of the angle that is formed by two tangent lines drawn to the circle. Solution

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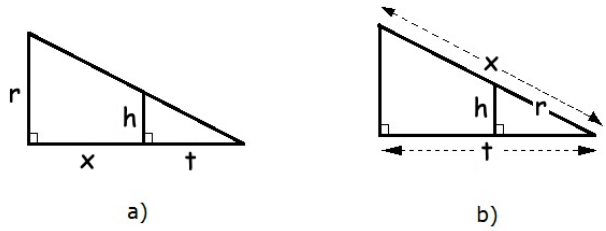
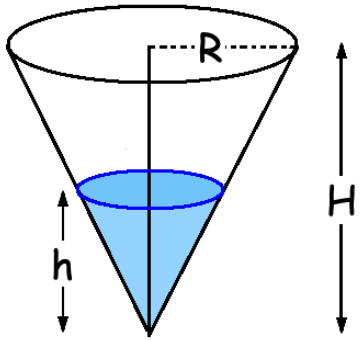
7 Calculus

The problems here are not calculus problem, they are just part of the review at the beginning of the course.

1. Express x in terms of the other variables in the picture. Solution



2. A water tank is of the shape of a cone . The tank is $H = 20\text{ft}$ tall and the circular top edge of the cone has a radius $R = 5\text{ft}$ long. If the surface of the water is at a height of $h = 3\text{ft}$, what is the surface area of the top of the water? Solution



3. Compute the given limits.
   a) $\lim_{x\to\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$
   b) $\lim_{x\to-\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$ Solution

4. Find all relative extrema for the function $f(x) = 2x^3 + 6x^2 - 54x - 15$. State the intervals over which f is increasing and decreasing. Solution

5. Find the slope of the tangent line drawn to the graph of $y = -\dfrac{1}{2}x^2 + 3x + 1$ from the point $P(4, 7)$. Solution

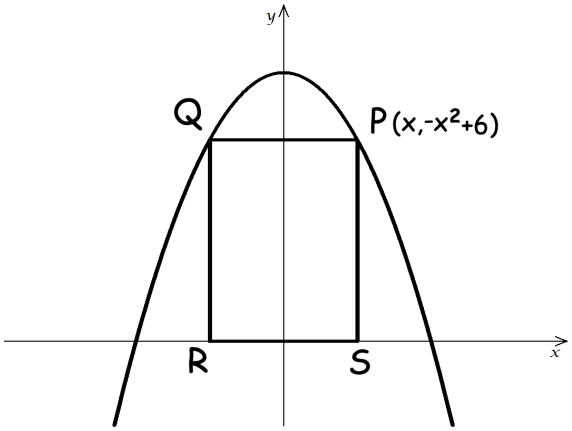
6. Find all values of b and c so that $y = 10x+1$ is tangent to $f(x) = x^3 + bx^2 + cx - 3$. Solution

7. Find an equation of the common tangent lines drawn to the graphs of $y = x^2$ and $y = (x-2)^2 + 12$. Solution

8. Find all values of p so that $y = 4x + 2$ is tangent to $y = px^2 + 8$. Solution

9. Find the greatest possible value of the product of two positive numbers if the sum of one number and the square of the other is 6. Solution

10. Let $P(x, y)$ be a point on the graph of $y = -x^2 + 6$ with $0 < x < \sqrt{6}$. Let PQRS be a rectangle with one side on the xaxis and two vertices on the graph, as shown on the picture. Find the exact value of the greatest possible area of such a rectangle. Solution



11. Find the point(s) on the graph of $y = 5x^2$ that is closest to the point $P(0, 2)$. Solution

12. Which square-based open box with a surface of 2400 square inches has the greatest volume? Solution

13. Compute the given limit. You may assume that $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = e$ and $\lim_{x\to-\infty}\left(1+\dfrac{1}{x}\right)^x = e$
    $\lim_{x\to\infty}\left(1+\dfrac{3}{x}\right)^x$ Solution

14. Compute each of the given indefinite integrals.
    a) $\int e^{3x-2}dx$
    b) $\int (5x+8)^{10}\,dx$
    c) $\int \cos(2x-\pi)\,dx$
    d) $\int \sin x \cos x dx$
    e) $\int \cos^2 x dx$
    f) $\int \dfrac{1}{3-x}dx$
    g) $\int \dfrac{6x-1}{2x+5}dx$
    Solution

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Average Speed and Velocity
Average Velocity

Complete Analysis of a Function - Part 1

Limits at Infinity - Part 1and Part 2

Two-Sided Limits
The First Derivative Test
Proving the Intermediate Value Theorem
The Bolzano-Weierstrass Theorem
A Definition of e

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