

Problem 1 Two mathematicians are having a conversation. Mathematician A asks B about his kids. B answers: "I have three children, the product of their ages is 36." A says: "I still don't know how old your children are." Then B tells A the sum of his three kids' ages. A answers: "I still don't know how old they are. Then B adds: "The youngest one has red hair." Now A knows how old the kids are. Do you?

Solution 1 The first information we have is that the product of the three kids' ages is 36. This gives us all possibilities. We need to find all possible ways we can express 36 as a product of 3 integers. Let us fix the age of the youngest kid.

If the youngest kid's age is 1, we have to express $\frac{36}{1} = 36$ as a product of two integers.

36

1	36
2	18
3	12
4	9
6	6

Thus we have so far:

youngest	middle	oldest
1	1	36
1	2	18
1	3	12
1	4	9
1	6	6

If the youngest kid's age is 2, we have to express $\frac{36}{2} = 18$ as a product of two integers.

18

1	18
2	9
3	6

Notice that the first row is repetition of a previously found case. The youngest kid being 2 years old mean the middle one cannot be 1 year.

Thus we have:

youngest	middle	oldest
2	2	9
2	3	6

If the youngest kid's age is 3, we have to express $\frac{36}{3} = 12$ as a product of two integers, where the smaller one is at least 3.

12

3	4
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Thus we have:

youngest	middle	oldest
3	3	4

Higher numbers will give us only repetition of previous cases. How do we know?....Thus we have so far:

<i>youngest</i>	<i>middle</i>	<i>oldest</i>
1	1	36
1	2	18
1	3	12
1	4	9
1	6	6
2	2	9
2	3	6
3	3	4

B tells A the sum of the kid's ages. We can add all these tripples.

<i>youngest</i>	<i>middle</i>	<i>oldest</i>	<i>sum of ages</i>
1	1	36	38
1	2	18	21
1	3	12	16
1	4	9	14
1	6	6	13
2	2	9	13
2	3	6	11
3	3	4	10

After B told A the sum of the ages, and A still didn't know the ages. This means that the sum was NOT ENOUGH information to distinguish two possibilities. Since the only sum that appears twice is 13, with 1, 6, 6 and 2, 2, 9, these are the only possibilities. (Any other sum was mentioned, A would have known the answer.) Since B said the YOUNGEST one has red hair, there is a youngest kid. This rules out 2, 2, 9 and so the kids are 1 and 6 and 6 years old.