

Puzzle 11 - SOLUTION

Problem 1 A bus makes a roundtrip between towns A and B. From A to B, the bus completes the trip with an average speed of $60 \frac{\text{mi}}{\text{hr}}$. From B to A, it travels with an average velocity of $40 \frac{\text{mi}}{\text{hr}}$. What is the average speed of the bus for the entire roundtrip? (Hint: it is NOT the average of 40 and 60)

Solution 1 Suppose the towns are 120 miles apart. We can use any numbers, but 120 is 'nice' because it is divisible by 40 and 60.

The time it took to travel from A to B was

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{120 \text{ mi}}{60 \frac{\text{mi}}{\text{hr}}} = 2\text{hr}$$

The trip from B to A:

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{120 \text{ mi}}{40 \frac{\text{mi}}{\text{hr}}} = 3\text{hr}$$

The average velocity for the round trip is

$$v = \frac{\text{total distance}}{\text{total time}} = \frac{240 \text{ mi}}{5 \text{ hr}} = 48 \frac{\text{mi}}{\text{hr}}$$

Thus the average velocity is $48 \frac{\text{mi}}{\text{hr}}$.

Note: We can obtain these results without making up a distance. Denote it by s . Then the computation works like this.

The time it took to travel from A to B was

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{s}{60}$$

The trip from B to A:

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{s}{40}$$

The average velocity for the round trip is

$$v = \frac{\text{total distance}}{\text{total time}} = \frac{s + s}{\frac{s}{60} + \frac{s}{40}} = \frac{2s}{\frac{2s}{120} + \frac{3s}{120}} = \frac{2s}{\left(\frac{5s}{120}\right)} = 2s \cdot \frac{120}{5s} = \frac{240s}{5s} = 48$$

Thus the average velocity is $48 \frac{\text{mi}}{\text{hr}}$.