

# 1 Pre-Algebra

## Parentheses do Not Mean Multiplication

### The Order of Operations Agreement

1. a) In what order do we perform multiplications and divisions?

**Solution**

- b) In what order do we perform additions and subtractions? **Solution**

2. Perform the given operations.

a)  $(29 - 11) + 4$

b)  $29 - (11 + 4)$

c)  $29 - 11 + 4$  **Solution**

3. Perform the given operations.

a)  $40 \div (5 \cdot 2)$

b)  $(40 \div 5) \cdot 2$

c)  $40 \div 5 \cdot 2$  **Solution**

4. Perform the given operations.

a)  $3^2 + 4^2$

b)  $(3 + 4)^2$  **Solution**

5. Simplify each of the given expressions.

a)  $2 + 3(4 + 5)$

b)  $(2 + 3)(4 + 5)$  **Solution**

6. Simplify each of the given expressions.

a)  $14 - 4(3^2 - 7)$

b)  $(14 - 4)(3^2 - 7)$  **Solution**

7. Simplify each of the given expressions.

a)  $67 - 4(5 + 3(4 \cdot 7 - 5^2))$  **Solution**

b)  $\frac{24 \div 4 \cdot 3}{9 - 4 + 1}$  **Solution**

c)  $8 + 12 \div 3 + 3 \cdot 2^2$  **Solution**

d)  $2(7 + 3(2 \cdot 8 - 5))$  **Solution**

e)  $2^4 + \frac{41 + 7}{2 \cdot 3} \div 4 - 4$  **Solution**

f)  $2^2 - 3(4 - 2(4 \cdot 3 - 5^2))$  **Solution**

g)  $7 - 4(5 - (-4)^2 + (-1)) + 1$  **Solution**

h)  $((3^2 - 5)^2 - 14)^2 - 1^3$  **Solution**

### Perimeter and Area of Right Triangles

8. Simplify each of the given expressions.

a)  $48 - \frac{6^2}{3}$

b)  $\frac{48 - 6^2}{3}$  **Solution**

### Divisibility

9. Consider the numbers

159, 289, 292, 444, 591, 50 020

Find all numbers from the list that are

a) divisible by 3

b) divisible by 4

c) divisible by 12 **Solution**

10. List all factors of 75. **Solution**

11. a) Is 57 a prime number?

b) Is 59 a prime number? **Solution**

12. Consider the numbers 21, 37, 77, 89, 93. Find all prime numbers from this list. **Solution**

13. Is 1139 a prime number? **Solution**

14. Find the prime factorization of 2250. **Solution**

15. True or false?

a) For any integer  $n$ , if  $n$  is divisible by 6 and by 7, then it is also divisible by 42.

b) For any integer  $n$ , if  $n$  is divisible by 6 and by 8, then it is also divisible by 48. **Solution**

16. Find the greatest common factor and least common multiple for the given pair of numbers.

a) 36 and 45 **Solution**

b) 189 and 126 **Solution**

17. Suppose that  $x$  is a positive integer, not 4 and not 200.  
Find all possible values of  $x$  so that  
 $\gcd(4, 200, x) = 4$  and  $\text{lcm}(4, 200, x) = 200$ .

[Solution](#)

18. Perform the given divisions with remainder.

a)  $137 \div 4$  [Solution](#)

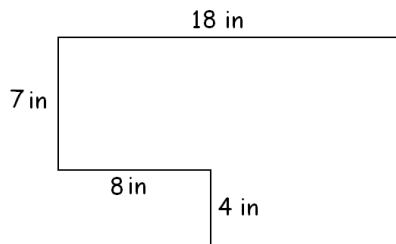
b)  $325 \div 7$  [Solution](#)

19. What is the last digit of  $2^{99}$ ? [Solution](#)

20. Find the perimeter and area of the rectangle with sides  
11 ft and 9 ft long. [Solution](#)

21. Find the perimeter and area of the right triangle with  
sides 10 in, 24 in, and 26 in long. [Solution](#)

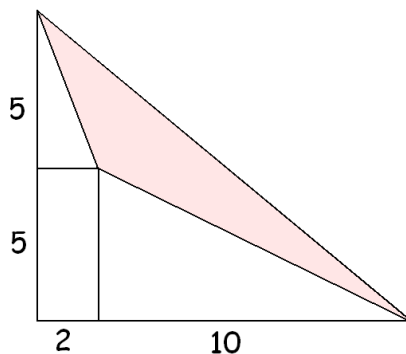
22. Find the perimeter and area of the figure shown.  
[Solution](#)



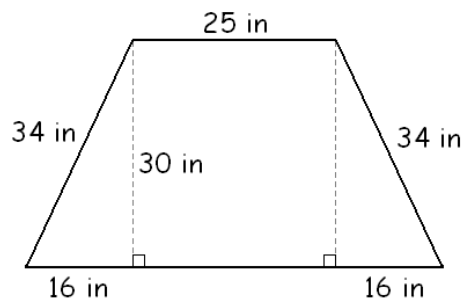
23. Suppose  $A$  is the set of all integers divisible by 5 and  
 $B$  is the set of all integers divisible by 10. Label each  
of the given statements true or false.

$A \subseteq B$     $B \subseteq A$    [Solution](#)

24. Find the area of the shaded region. [Solution](#)



25. Find the perimeter and area of the trapezoid shown on  
the picture. [Solution](#)

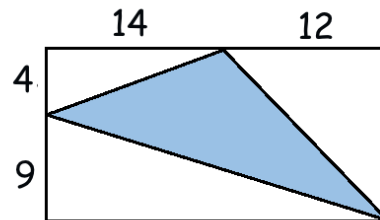


26. Plot the given points and compute the perimeter and  
area of the figure you got.  $A(-1, -2)$ ,  $B(1, -2)$ ,  
 $C(1, 3)$ , and  $D(-1, 3)$  [Solution](#)

27. Compute the area of the parallelogram given by  
the points  $A(-4, -1)$ ,  $B(3, -1)$ ,  $C(5, 3)$ , and  
 $D(-2, 3)$ . [Solution](#)

28. Compute the area of the triangle determined by  
 $A(-5, 2)$ ,  $B(-1, 2)$ , and  $C(-1, -3)$ . [Solution](#)

29. Find the area of the shaded region. [Solution](#)



30. Simplify each of the following.

a)  $5^2 - 9^2$

b)  $(5 - 9)^2$  [Solution](#)

31. Simplify each of the following.

a)  $|8 - 15|$

b)  $|8| - |15|$  [Solution](#)

32. Simplify each of the following.

a)  $\sqrt{25} + \sqrt{144}$

b)  $\sqrt{25 + 144}$  [Solution](#)

33. Simplify  $\sqrt{29} - \sqrt{16}$  [Solution](#)

34. Simplify  $7 - 2(-3)$  [Solution](#)

35. Simplify each of the following.

a)  $11 - 5(4 - 7)$  [Solution](#)

36. Simplify each of the following.

a)  $-3^3$     b)  $(-3)^2$     c)  $-(-3)^2$

**Solution**

37. Simplify each of the following.

a)  $4 - (-5) + 7$

b)  $(4 - (-5)) + 7$

c)  $4 - ((-5) + 7)$     **Solution**

38. Simplify each of the given expressions.

a)  $-5 - 7(-4 - 6)$

b)  $-(5 - 7(-4 - 6))$

c)  $-(5 - 7(-4)) - 6$

d)  $(-5 - 7)(-4 - 6)$

e)  $-(5 - 7(-4) - 6)$

f)  $-(5 - 7 - (4 - 6))$

g)  $-5 - 7 - (4 - 6)$

h)  $-(5 - 7)(-4 - 6)$

i)  $-(5 - 7) - 4(-6)$

j)  $-5(-7) - 4(-6)$     **Solution**

39. Simplify each of the given expressions.

a)  $-((-3) + (5 - 1)) - 7^2$     **Solution**

b)  $\frac{5^2 - 3^2}{14 - 3 \cdot 2^2 - 2}$     **Solution**

c)  $-2(-1 + 10(7 - 5)) - (-7)^2$     **Solution**

d)  $\sqrt{(2 - (-4))(9 - 3 \div 3) - (-1)^3}$     **Solution**

40. Simplify each of the following.

a)  $|-3 - 8| - |11 + 9|$

b)  $|-3 - 8 - |11 + 9||$

c)  $|-3 - 8|-11 + 9||$

d)  $|-3 - |8 - 11 + 9||$

e)  $|-3|-8 - 11 + 9||$     **Solution**

41. Simplify each of the following.

a)  $|-12 - 7| - |1 + 11|$

b)  $|-12 - 7|-1 + 11||$

c)  $|-12|-7 - 1 + 11||$     **Solution**

42. Evaluate each of the following.

a)  $3a^2 - 2b + 4$  if  $a = 2$  and  $b = 5$

b)  $3a^2 - 2b + 4$  if  $a = 5$  and  $b = 2$

c)  $3x^2 - 5x + 8$  if  $x = 3$     **Solution**

43. Evaluate each of the following.

a)  $\frac{n - 4}{k}$  if  $k = -1$  and  $n = 7$ .    **Solution**

b)  $-x^2 + 3x - 5$  if  $x = -8$     **Solution**

c)  $11a + 4b - 2c$  if  $a = 5$ ,  $b = 3$ , and  $c = -2$

**Solution**

d)  $\frac{9 - (6a + b)}{ab + 10}$  if  $a = -2$  and  $b = 5$     **Solution**

e)  $2x^2 - 3y^3$  if  $x = 5$  and  $y = -2$     **Solution**

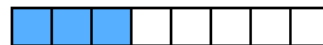
44. Given the values  $-4, -2$ , and  $6$ , find all numbers listed that are solution of the equation

$x^3 - x^2 - 27x + 2 = x^2 - 3x + 2$ .    **Solution**

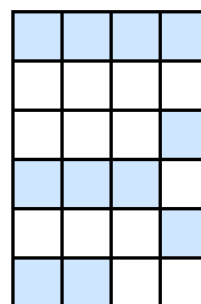
45. Is the statement  $9x^2 + 9x + 3 < 2x + 104$  true or false when  $x = 3$ ?    **Solution**

46. Express the shaded area as a fraction.

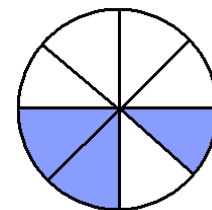
a)    **Solution**



b)    **Solution**



c)    **Solution**



47. Find each of the following.

a)  $\frac{1}{6}$  of 18

b)  $\frac{5}{6}$  of 18    **Solution**

48. Find 9% of 200.    **Solution**

49. In a class, there are 29 students. 9 of the students are men. What fraction of the students are men?

**Solution**

50. There are 800 students enrolled in a school. If  $\frac{3}{4}$  of the students are boys, how many students are girls?  
[Solution](#)
51. A \$500 TV went on a 6% off sale. What is the sale price?  
[Solution](#)
52. a) Rewrite 48% as a reduced fraction. [Solution](#)  
b) Re-write  $\frac{4}{5}$  with a denominator 10. [Solution](#)
53. In my left pocket, I have 11 dollars and 87 cents. In my right pocket, I have 2 dollars and 65 cents. How much money do I have all together?  
[Solution](#)
54. Bring  $\frac{5}{11}$  and  $\frac{4}{9}$  to to a common denominator in order to compare the two fractions. Use the least common denominator. [Solution](#)
55. Re-write as percent:  
a)  $\frac{19}{25}$     b)  $\frac{3}{5}$     [Solution](#)
56. a) If the price went up from \$1200 to \$1560, what percentage of a change is this?  
b) 1560 is what percent of 1200? [Solution](#)
57. a) 80% of a number is 72. Find this number.  
[Solution](#)  
b) 125% of a number is 450. Find this number.  
[Solution](#)
58. We decreased quantity Q by 5%. Which of the given expressions does that?  
0.5Q    0.05Q    95Q    0.95Q    1.05Q    [Solution](#)
59. Which of the following expresses an 80% increase of quantity Q.  
A) 9Q                      C) 1.08Q                  E) 0.8Q  
B) 80Q                     D) 1.8Q                    F) 180Q  
[Solution - Youtube link](#)
60. Express each of the given expressions as a percentage change from quantity Q. [Solution](#)  
1.8Q                      1.08Q                      0.92Q  
8Q                         9Q                            0.2Q
61. 125% of a number is 450. Find this number.  
[Solution](#)
62. a) We increased \$3000 by first 20% and then later by another 30%. What is the resulting number? Express the two changes as a single change. What percentage is this change? [Solution](#)  
b) We increased a quantity by 25% and then later decreased it by 8%. Express the two changes as a single change. What percentage is this change? [Solution](#)
63. a) A TV went on a 15% off sale. The sale price is \$238. What was the original price? [Solution](#)  
b) Jake got an 8% raise. Now he is making \$3510 a month. How much was he making before the raise?  
[Solution](#)
64. Simplify each of the following.  
a)  $\left(\frac{1}{5} - \frac{1}{7}\right) \div \frac{24}{35}$     [Solution](#)  
b)  $-\frac{9}{10} - \left(-\frac{3}{4}\right)$     [Solution](#)  
c)  $\left(-\frac{1}{2}\right)^2 - \frac{3}{10}$     [Solution](#)  
d)  $2 - \frac{3}{4} \cdot \frac{1}{6}$     [Solution](#)
65. Evaluate  $-x^2 - 2x + 3$  if  $x = -\frac{3}{5}$   
[Solution - Youtube link](#)
66. Verify that the repeating decimal  $1.174747474\dots = 1.1\overline{74}$  represents a rational number by converting it into a fraction of two integers.  
[Solution](#)
67. Solve each of the given equations.  
a)  $\frac{x + 12}{5} = 6$     [Solution](#)  
b)  $\frac{x - 4}{3} = 5$     [Solution](#)
68. Solve each of the given equations.  
a)  $3x + 8 = -7$     [Solution](#)  
b)  $-5x + 3 = 38$     [Solution](#)  
c)  $\frac{x + 20}{3} = 2$     [Solution](#)  
d)  $\frac{x + 3}{-5} = 2$     [Solution](#)  
e)  $\frac{x - 2}{5} = -1$     [Solution](#)

f)  $\frac{x}{-4} + 5 = 8$       [Solution](#)

69. Solve each of the given equations.

a)  $-\frac{2}{3}x - \frac{1}{8} = \frac{13}{24}$       [Solution](#)

b)  $\frac{q + \frac{4}{5}}{\frac{3}{2}} = -\frac{1}{15}$       [Solution](#)

70. Ben took five exams. Four of the exams were 71, 75, 67 and 93 points. What was the score on the fifth exam if the average of all five exams was 78 points.

[Solution - Youtube link](#)

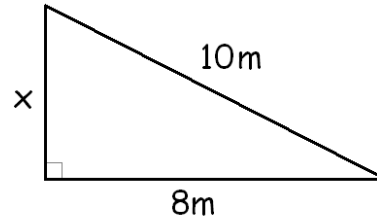
**Weighted Averages**

71. Prove that the repeating decimal 0.78257257257... is

rational by re-writing it as a quotient of two integers. You do not need to reduce the fraction to lowest terms.

[Solution](#)

72. Find the missing side in the right triangle shown on the picture if we know that the area of the triangle is 24 m<sup>2</sup>. [Solution](#)



## 2 Introductory Algebra

**The Words AND and OR**

1. Determine whether the statements given are true or false.

a)  $3 \leq 3$     b)  $5 \leq 3$     c)  $2 \leq 10$       [Solution](#)

2. Determine whether the given statements are true or false.

- a) 34 is not divisible by 7, or 39 is divisible by 9.
- b) 34 is not divisible by 7, and 39 is divisible by 9.

[Solution](#)

3. Determine whether the given statements are true or false.

- a) Seven is less than nine, and 5 is odd.
- b) Six is a natural number and zero is a natural number.
- c) Four is less than five, and five is even.      [Solution](#)

4. Determine whether the given statement is true or false. *The empty set is a subset of any set, or the number 11 is even.* [Solution](#)

5. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers  $x$  from this list such that

- a)  $x \geq 6$  and  $x$  is odd
- b)  $x \geq 6$  or  $x$  is odd      [Solution](#)

6. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers  $x$  from this list such that

- a)  $x < 7$  or  $x > 3$
- b)  $x < 7$  and  $x > 3$       [Solution](#)

7. Given the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Find all numbers  $x$  from this list such that

- a)  $x < 7$  and  $x < 4$
- b)  $x < 7$  or  $x < 4$       [Solution](#)

**Introduction to Set Theory**

8. Suppose that  $A = \{1, 5, 6, 8, 10\}$  and  $B = \{5, 6, 10\}$ . Determine whether the given statements are true or false.

a)  $4 \in A$       b)  $B \subseteq A$       [Solution](#)

9. Which statement is true?

$\{1, 3, 6\} \subseteq \{1, 2, 3, 4\}$   
 $\{1, 3, 6\} \not\subseteq \{1, 2, 3, 4\}$       [Solution](#)

10. Which statement(s) are true?

$$\{b, c, d, e\} \subseteq \{a, b, c, d, e, f, g, h\}$$

$$\{b, c, d, e\} \not\subseteq \{a, b, c, d, e, f, g, h\} \quad \text{Solution}$$

11. Which statement is true?

$$\{2, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\{2, 4, 6\} \not\subseteq \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \quad \text{Solution}$$

12. Which statement is true?

$$\{-1, 0, 2\} \subseteq \mathbb{N}$$

$$\{-1, 0, 2\} \not\subseteq \mathbb{N} \quad \text{Solution}$$

13. True or false?

$$\{x \mid x \text{ is a cat}\} \subseteq \{x \mid x \text{ is a black cat}\}$$

$$\{x \mid x \text{ is a cat}\} \not\subseteq \{x \mid x \text{ is a black cat}\} \quad \text{Solution}$$

14. Suppose that  $E$  is the set of all even numbers and  $O$  is the set of all odd numbers. Which of the given statements are true?

$$E \subseteq O \quad O \subseteq E \quad \text{Solution}$$

15. Two sides of a triangle are 5 units and 13 units long.

a) What lengths are possible for the third side?

**Solution**

b) What lengths are possible for the third side if it is to be the longest side? **Solution**

c) What lengths are possible for the third side if it is to be the shortest side? **Solution**

16. Simplify each of the following.

a)  $(2p - 3) + (-5p + 6)$

b)  $(2p - 3) - (-5p + 6)$

c)  $2(2p - 3) - 5(-5p + 6)$

**Solution**

17. Three sides of a triangle are different integers. What is the smallest possible value of the perimeter of this triangle? **Solution**

18. Simplify each of the following.

a)  $(2x + 11y + 3) + (20x - 11y + 9)$  **Solution**

b)  $4(4x - 3) - 3(-x + 2)$  **Solution**

c)  $-5(3a + 4(7a + 3(-3a - 4)))$  **Solution**

d)  $-2(6x - 3(3x - 2(2x - 5)))$  **Solution**

e)  $\frac{q + 7}{8} - \frac{3q - 5}{3}$  **Solution**

f)  $(2x + 1)^2 + (2x - 1)^2$  **Solution**

g)  $(3x + 5)^2 - (3x - 5)^2$  **Solution**

h)  $(2x - 1)^2 - (3x + 2)(4x - 3)$  **Solution**

i)  $(3x^3 - 2)^2$  **Solution**

j)  $(2x^4 - 5y^3)^2$  **Solution**

19. a) Jake studies  $x$  hours for the test. Julie studied twice as long. Express the time, in terms of  $x$ , that Julie spent studying. **Solution**

b) Ann spent 18 dollars less than Juan. If Juan's spending is denote by  $x$ , express Ann's spending in terms of  $x$ . **Solution**

c) We purchased  $y$  many posters, each for \$4. Write an algebraic expression for the amount of money we spent. **Solution**

d) A roof's length is 7 feet less than twice its width. If we let  $t$  represent the width, write an algebraic expression for the length. **Solution**

20. Translate the given expression.

The sum of a number increased by one and the number decreased by six. **Solution**

21. Voltage taken by resistor 1 is ten more than four times the voltage taken by resistor 2. Express the voltage taken by resistor 1 in terms of the voltage taken by resistor 2. Use  $x$  for the variable. **Solution**

22. Translate the given sentence to an equation using  $x$  for the number.

a) The sum of three times a number and six is nine. **Solution**

b) The sum of a number and four is ten. **Solution**

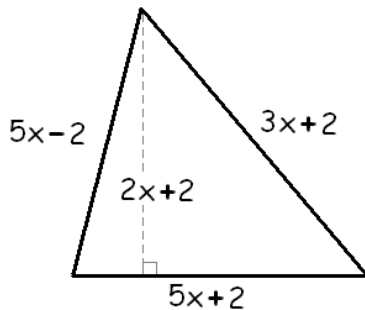
c) The product of three more than a number and eight is 48. **Solution**

23. Translate the given sentence to an equation using  $x$  for the number. Then solve the equation for  $x$ .

a) The sum of a number and four is ten. Find this number. **Solution**

- b) The sum of three times a number and six is nine. Find this number. [Solution](#)
- c) The product of three more than a number and eight is 48. Find this number. [Solution](#)

24. Express the perimeter of the triangle in terms of  $x$ .  
[Solution](#)



25. Solve each of the given equations.

- b)  $\frac{x-5}{-4} + 5 - 1 = 4$  [Solution](#)
- c)  $4(3x+1) - (x+3) = 10x - 4$  [Solution](#)
- d)  $\frac{x+5}{7} - \frac{2x+1}{9} = x+6$  [Solution](#)
- e)  $\frac{1}{10}(x-14) = \frac{5}{2}x - \frac{3}{5}$  [Solution](#)
- f)  $-3(x+4(x+3(-3x-1))) = -3+7(8x-5)$   
[Solution](#)
- g)  $-5(x+2(x-2(-4x+3))) = 60(-5x+1)$   
[Solution](#)
- h)  $(x+2)(2x-3) = 2(x+1)^2 - 17$  [Solution](#)

26. Solve each of the given inequalities.

- a)  $\frac{3x+6}{-4} \geq 1$  [Solution](#)
- b)  $4(3x-7) - 5(2x-5) > -(-3x+2) + 1$   
[Solution](#)
- c)  $\frac{x-4}{5} - \frac{5x-1}{2} > -2x-3$  [Solution](#)

27. There were 7 more women than men at the party. If there were 31 guests, how many of them were women? [Solution](#)

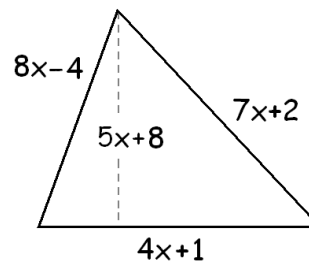
28. Susan is asked about her age. She answers as follows. "My age is 8 years less than twice the age of my brother." How old is her brother if Susan is 24 years old? [Solution](#)

29. The cost of a taxi ride is \$1.55 for the first mile and \$1.35 for each additional mile or part thereof. Find the distance we covered if we paid \$24.50. [Solution](#)

30. Wendy is asked about her age. She answers as follows. "My age is seven years less than twice the age of my son. The sum of our ages is 61 years. How old is Wendy?" [Solution](#)

31. The first night in a hotel costs 55 dollars. All additional nights costs 42 dollars. How many nights did we spend in the hotel if our bill was 265 dollars?  
[Solution](#)

32. Find the value of  $x$  if the triangle shown on the picture has a perimeter of 189 units. [Solution](#)



33. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost 11 dollars, adult tickets cost 14 dollars. The number of children tickets purchased was five less than twice the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of 465 dollars? [Solution](#)

34. The area of a square would increase by 17 square-yards if we increased its sides by 1 yard. How long is a side of the square now (before the increase)?  
[Solution](#)

35. Five times the sum of four and a number is six less than seven times the same number. [Solution](#)

36. Suppose that  $U = \{1, 2, 3, \dots, 10\}$ ,  $A = \{2, 6, 9, 10\}$ , and  $B = \{1, 2, 3, 7, 10\}$ . Find  $A \cap B$ .  
[Solution](#)

37. Suppose that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{x \in U : x \leq 2\}$ , and  $B = \{x \in U : x \geq 7\}$ . Find each of the following sets.

- a)  $A \cup B$       b)  $A \cap B$  [Solution](#)

38. Suppose that  $U = \{1, 2, 3, \dots, 10\}$ ,  $A = \{1, 3, 5, 7\}$ , and  $B = \{1, 2, 3, 4\}$ . Compute  $B \cap$

39. Suppose that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{x \in U : x < 8\}$ , and  $B = \{x \in U : x > 4\}$ . Find each of the following sets.

a)  $A \cup B$       b)  $A \cap B$       [Solution](#)

40. Suppose that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{2, 4, 6, 8, 9\}$ , and  $B = \{1, 2, 5, 6, 9\}$ . Find each of the following.

a)  $\overline{A \cap B}$       b)  $\overline{A \cup B}$       c)  $\overline{A \cup \overline{B}}$       [Solution](#)

41. Suppose that  $A$  and  $B$  are sets such that  $A \cap B = \{1, 2, 5\}$  and  $A \cup B = \{1, 2, 3, 4, 5, 6\}$ . How many sets are possible for  $A$ ?      [Solution](#)

42. State the coordinates of the points plotted.      [Solution](#)

43. Compute the area of the triangle determined by the points  $A(-5, 2)$ ,  $B(-1, 2)$ , and  $C(-1, -3)$ .      [Solution](#)

44. Find the exact value of  $x$  based on the picture.      [Solution](#)



45. Solve each of the given systems of equations.

a) [Solution](#)

$$\begin{aligned} x + 4y &= -22 \\ y &= 3x - 12 \end{aligned}$$

b) [Solution](#)

$$\begin{aligned} (x - 2)^2 + (y + 3)^2 &= x^2 - y(-1 - y) - 42 \\ (x + 1)(y - 1) &= xy - 14 \end{aligned}$$

46. Find the exact value of the distance between the points  $A(-6, 4)$  and  $B(6, -1)$ .      [Solution](#)

47. Graph each of the following.

a)  $y = \frac{2}{3}x + 1$       [Solution](#)

b)  $4x + 3y = -12$       [Solution](#)

c)  $3x - 4y = -8$       [Solution](#)

d)  $2x + 3y = -5$       [Solution](#)

48. Compute each of the given sums.

a)  $1 + 2 + 3 + \dots + 100$       [Solution](#)

b)  $3 + 6 + 9 + \dots + 2019$       [Solution](#)

c)  $30 + 37 + 44 + \dots + 2018$       [Solution](#)

d)  $\frac{1}{1001} + \frac{2}{1001} + \frac{3}{1001} + \dots + \frac{1000}{1001}$       [Solution](#)

49. The first row in a theater has 31 seats in it. The second row has one more seats than the first row. The third row has one more seats than the second row. And so on, each row has one more seats than the row before. If the last row has 191 seats in it, how many seats are there in the entire theater?      [Solution](#)

50. There is a farm where chickens and cows live. All together, there are 66 heads and 202 legs. How many chickens, how many cows?      [Solution](#)

51. We have 260 coins, all nickels and dimes, in the value of 20.25 dollars. How many of each?      [Solution](#)

52. Find the perimeter and area of the parallelogram determined by the points  $A(-2, -3)$ ,  $B(21, -3)$ ,  $C(5, 9)$ , and  $D(-18, 9)$ .      [Solution](#)

53. There are 85 men and 15 women in a large conference room.

a) How many handshakes would take place if all person shook hands with all other people in the room?

b) How many handshakes would take place if all men shook hands with all other men in the room, and all women shook hands with all women in the room, but no man shook hands with any woman?

c) How many handshakes would take place if all men shook hands with all woman, but no two men or two women shook hands?      [Solution](#)

54. Simplify each of the given expressions.

a)  $(3x + 5) + (-4x + 1)$

b)  $(3x + 5) - (-4x + 1)$

c)  $-2(3x + 5) - 3(-4x + 1)$

d)  $(3x + 5)(-4x + 1)$       [Solution](#)



55. Simplify each of the given expressions.

a)  $-(5x - 1)^2$  [Solution](#)

b)  $(2z^5 + 1)(2z^5 - 1)$  [Solution](#)

56. Simplify each of the given expressions.

a)  $(-3x^4)(-2x^5)$  [Solution](#)

b)  $\frac{(x^4)^6}{x^4x^6}$  [Solution](#)

c)  $\left(\frac{-12x^{10}y^4z^5}{15x^6y^4z^3}\right)^3$  [Solution](#)

d)  $\frac{(x^2y^5)^3(-x^4y^3)^6}{(-xy^4x^4)^5}$  [Solution](#)

e)  $\frac{-18(-x)^{10}(y^2)^8}{24x^2(-y)^5x^5}$  [Solution](#)

57. Solve each of the following.

a)  $4(x + 1)(x - 2) = 0$

b)  $x(x + 1)(x - 2) = 0$  [Solution](#)

58. Solve each of the following.

a)  $(x + 7)(x - 8) = 0$  [Solution](#)

b)  $-2x(3x + 1) = 0$  [Solution](#)

c)  $-x^6 = 3x^5$  [Solution](#)

d)  $6x^7 = 3x^6$  [Solution](#)

e)  $x^6 = 16x^4$  [Solution](#)

f)  $x^2 - 2x = 0$  [Solution](#)

59. Convert 50 202 000 to scientific notation. [Solution](#)

60. Perform the given multiplication. Present your answer in scientific notation.

$(1.2 \cdot 10^5)(8.5 \cdot 10^7)$  [Solution](#)

61. If we increase one side of a square by 1 unit and decrease the other side by 5 units, the resulting rectangle's area is 77 square-unit less than the area of the original square. How long was the side of the original square? [Solution](#)

62. Factor out the GCF (greatest common factor) in  $35x^7 + 10x^6 + 55x^4$ . [Solution](#)

63. Factor out  $-1$  in the given expression  $-6x^5 + x^3 - 5$  [Solution](#)

64. Completely factor each of the following.

a)  $12x + 75$

b)  $12x^2 + 75$

c)  $12x^2 - 75$  [Solution](#)

65. Completely factor each of the following.

a)  $3y^5 - 12y^3$  [Solution](#)

b)  $-2m^4 + 32$ . [Solution](#)

c)  $-98x^4 + 50$  [Solution](#)

d)  $(2x^2 + 3x - 4)^2 - (2x^2 - 3x + 5)^2$  [Solution](#)

66. Solve each of the given equations.

a)  $x^2 - 2x = 0$  [Solution](#)

b)  $x^6 = 16x^4$  [Solution](#)

c)  $6x^7 = 3x^6$  [Solution](#)

d)  $(x - 2)^{12}(x + 6)^6 + (x - 2)^{11}(x + 6)^7 = 0$  [Solution](#)

67. Simplify each of the given expressions.

a)  $\frac{2 - 7x}{7x - 2}$  [Solution](#)

68. If we raise a number  $x$  to the third power, the result is the same as 36 times the number. Find all possible values of  $x$ . [Solution](#)

69. Find all numbers whose square is seven greater than six times the number. [Solution](#)

70. Find the prime factorization of  $12^{50}$ . [Solution](#)

71. Suppose that  $A = 3^{500}$ . Express each of the following in terms of  $A$ .

a)  $3^{501}$

b)  $6 \cdot 3^{500} - 3^{502}$

c)  $3^{499}$

d)  $9^{500}$

e)  $3^{1500}$  [Solution](#)

72. Simplify the given expression. [Solution](#)

$$\frac{4^{63} - 4^{60}}{4^{60}}$$

73. Simplify each of the given expressions. Present your answer using only positive exponents.

a)  $\frac{3}{x^{-5}}$

c)  $\frac{x^7}{3x^{-4}}$

b)  $\left(\frac{24}{8x}\right)^{-2}$

d)  $\frac{x^{-4}}{x^{-7}}$  **Solution**

74. Simplify each of the given expressions. Present your answer using only positive exponents.

a)  $\frac{x^{-7}y^{-2}}{x^{-4}y^{-6}}$  **Solution**

b)  $\frac{(x^{-4})^5}{(x^3)^{-8}}$  **Solution**

c)  $\frac{12x^{-7}(y^2)^{-5}}{15xy^{-3}x^2}$  **Solution**

d)  $\frac{-10(-a)^6(b^{-1})^3}{8a^{-3}(-b)^7a^5(-b)^{-4}}$  **Solution**

e)  $\frac{(x^6y^{-2})^3(-x^5y^3)^{-2}}{(-x^2y^2x)^{-3}}$  **Solution**

f)  $\frac{3^{-1} + 5^{-1}}{3^{-1} - 5^{-1}}$  **Solution**

g)  $6^{-1} - 5 \cdot 2^{-3}$  **Solution**

h)  $\frac{16 - x^{-2}}{4 + x^{-1}}$  **Solution**

75. Rewrite the given set using interval notation. **Solution**

$$\{x : x \leq -7 \text{ or } x > -1\}$$

76. Find all integer values of  $n$  for which  $\frac{3n+2}{n-4}$  is an integer. **Solution**

### 3 Intermediate Algebra

Completing the Square **Part 0**, **Part 1**

**Smallest Value of a Quadratic Expression - 1**

1. Factor each of the following completely by completing the square.

a)  $-3a^2 + 12a + 15$  **Solution**

b)  $-x^2 + 2x - 1$  **Solution**

c)  $-2x^2 + 12x - 50$  **Solution**

2. Solve each of the given equations by completing the square.

a)  $-3x^2 + 24x - 36 = 0$  **Solution**

b)  $3x^2 - 6x + 3 = 0$  **Solution**

c)  $-2x^2 + 8x - 10 = 0$  **Solution**

d)  $(x - 8)(x - 6) = 24$  **Solution**

e)  $3x^2 + 30x = -66$  **Solution**

f)  $15x^2 + x - 2 = 0$  **Solution**

3. Factor by completing the square.

a)  $x^2 - 3x - 10$  **Solution**

4. Find the smallest value of each of the following expressions.

a)  $x^2 - 10x + 14$

b)  $(2a - 1)^2 + 8$

c)  $x^2 - 8x - 180$  **Solution**

5. Graph  $y = x^2 - 4x - 5$ , State the coordinates of at least five points, including vertex and intercepts.

**Solution**

6. Simplify each of the given expressions.

a)  $\frac{x^3 - 16x}{x^2 + 6x + 8}$  **Solution**

b)  $\frac{x^2 + 8x + 7}{x^3 + 6x^2 + 5x}$  **Solution**

7. Match the expressions with the descriptions.

i)  $(x + 6)^2$                       iii)  $(x + 6)^2 + 25$

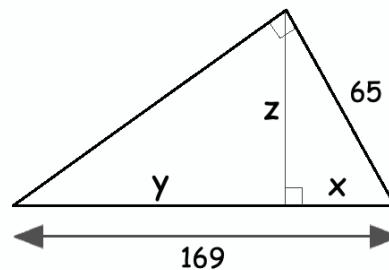
ii)  $(x + 6)^2 - 4$               iv)  $(x + 6)^2 - 3$

**Solution**

1) The expression can be factored over the real numbers but not over the integers.

- 2) The expression can be factored over the integers into two different linear factors.
- 3) The expression can not be factored over the integers or over the real numbers.
- 4) The expression is factored over the integers into two identical factors.
8. One side of a rectangle is 12 inches shorter than three times another side. Find the sides of the rectangle if its area is 1575 square-inches.  
**Solution**
9. Find all numbers whose square is seven greater than six times the number. **Solution**
10. We are standing on the top of a 720 ft tall building and throw a small object upward. The object's distance from the ground, measured in feet, after  $t$  seconds is  $-16t^2 + 128t + 1344$ . How long until the object hits the ground? **Solution**
11. Mia was asked about her age. Her answer was: my sister's age is ten years less than three times my age. The product of our ages is 312. How old are the sisters? **Solution**
12. We are organizing a fundraising event. We find that if we set the price at \$40, then we can sell 350 tickets. For each dollar increase in the price, five less tickets will be sold.
- a) How much money will we raise from the ticket sales if we set the price of a ticket to be \$50?
- b) What price(s) will result in a total income of \$14 625? **Solution**
13. Simplify each of the following.
- a)  $\sqrt{100}$                       e)  $\sqrt[3]{8}$                       **Solution**  
 b)  $-\sqrt{100}$                       f)  $-\sqrt[3]{8}$   
 c)  $\sqrt{-100}$                       g)  $\sqrt[3]{-8}$   
 d)  $-\sqrt{-100}$                       h)  $-\sqrt[3]{-8}$
14. Simplify each of the following.
- a)  $(\sqrt[5]{-3})^{20}$   
 b)  $(\sqrt[4]{-3})^{20}$                       **Solution**  
 c)  $\sqrt{x^{16}}$                       **Solution**
15. Simplify each of the given expressions.
- a)  $\sqrt{90} - 3\sqrt{40} + \sqrt{10}$                       **Solution**  
 b)  $(5\sqrt{2} - 1)(3\sqrt{2} + 5)$                       **Solution**  
 c)  $(4\sqrt{2} - 5)^2$                       **Solution**  
 d)  $(3 - \sqrt{10})^2 - (3 - \sqrt{10})(3 + \sqrt{10})$   
**Solution**
16. Rationalize the denominator in each of the given fractions.
- a)  $\frac{6}{2 - \sqrt{7}}$                       **Solution**  
 b)  $\frac{\sqrt{5} + 3}{\sqrt{5} - 1}$                       **Solution**  
 c)  $\frac{4}{\sqrt{15} + \sqrt{5}}$                       **Solution**
17. Re-write  $\sqrt[3]{x^7}$  using rational exponents. **Solution**
18. Simplify each of the following.
- a)  $x^{1/7} \cdot x^{1/14}$                       c)  $(x^{5/8})^{2/15}$   
 b)  $\frac{x^{3/5}}{x^{2/3}}$                       d)  $\left(\frac{x^{7/5}}{x^{3/5}}\right)^{5/6}$                       **Solution**
19. Simplify each of the following.
- a)  $\left(\frac{a^{3/5}}{b^{1/3}}\right)^3 \left(\frac{b^{1/2}}{a^{5/6}}\right)$                       **Solution**
20. Use exponential notation to simplify
- a)  $\sqrt[20]{x^{12}}$ .                      **Solution**
21. Find the exact value of  $-2x^2 - x + 1$  if  $x = 3 - 2\sqrt{5}$   
**Solution**
22. Simplify each of the given expressions.
- a)  $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$                       **Solution**
23. Solve the given compound inequalities.
- a)  $-x + 2 \leq 6$  or  $-4x + 1 < -19$   
 b)  $-x + 2 \leq 6$  and  $-4x + 1 < -19$                       **Solution**
24. Solve each of the given equations.
- a)  $\frac{x}{x + 5} - \frac{x + 5}{x} = 0$                       **Solution**

25. One number is ten greater than another. What is the smallest possible value of their product? [Solution](#)
26. a) How many four-digit numbers can be formed using only the digits 1, 3, 4, 5, and 7?  
 b) How many four-digit numbers can be formed using only the digits 1, 3, 4, 5, and 7? if repetition of digits is not allowed? (i.e 1513 is not allowed) [Solution](#)
27. Suppose that  $D = \{1, 2, 3\}$ , and  $Y = \{a, b, c, d, e\}$ .  
 a) How many functions are possible with domain  $D$  and range  $R$ , where  $R$  is a non-empty subset of  $Y$ ?  
 b) How many one-to-one functions are possible with domain  $D$  and range  $R$ , where  $R$  is a non-empty subset of  $Y$ ?  
 c) How many relations are possible with domain  $D$  and range  $R$ , where  $R$  is a non-empty subset of  $Y$ ? [Solution](#)
28. Consider the triangle with sides 8, 16, and 17 units long. Is it a right triangle? [Solution](#)
29. Find the exact value of the distance between the points  $A(3, -6)$  and  $B(-3, 1)$ . [Solution](#)
30. Two sides of a right triangle are 9 and 40 units long. Find the exact value of the third side. [Solution](#)
31. The base of a straight pyramid is a square with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. [Solution](#)
32. The hypotenuse of a right triangle is 34 cm long. The difference between the other two sides is 14 cm. [Solution](#)
33. The shortest side of a right triangle is 24 miles long. The difference between the other two sides is 4 miles. Find the missing sides. [Solution](#)
34. The base of a straight pyramid is a square with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. [Solution](#)
35. Find the exact values of  $x$ ,  $y$ , and  $z$ . [Solution](#)



## 4 General Education Mathematics

Geometry 1

Part 1.: [Standard labeling, Sides and Angles](#)

Part 2: [The sum of the Inner Angles of triangles.](#)

Part 3. [Angles of Polygons](#)

[Introduction to Set Theory](#)

[More Set Operations](#)

[Introduction to Combinatorics](#)

[Fundamental Counting Principle and Permutations](#)

[Listing and Counting Subsets](#)

[More Listing](#)

[Introduction to Probabilities](#)

[Combinations](#)

1. Suppose that  $U = \{1, 2, 3, \dots, 12\}$ ,  $A = \{1, 2, 3, 7, 8\}$ ,  $B = \{3, 4, 5, 7, 8, 12\}$ , and  $C = \{2, 3, 4, 6, 7, 11, 12\}$ . Find each of the given sets.
  - a)  $(A \cup B) \setminus C$
  - b)  $A \cup (B \setminus C)$       [Solution](#)
  
2. 12 women and 8 men will compete in a race. What is the probability that the first three finishers are
  - a) all men
  - b) 2 men and 1 women?      [Solution](#)
  
3. We toss a coin ten times. What is the probability that the results will be
  - a) exactly 3 heads
  - b) at least 3 heads?      [Solution](#)
  
4. There are 7 blue and 3 red marbles in a bag. We randomly pull two marbles, with replacement. Find each of the probabilities.
  - a) Both marbles pulled are blue.
  - b) Both marbles pulled are red.
  - d) The marbles we pulled have the same color.
  - e) The marbles we pulled are of different colors.
  - f) g) h) i) Find the same probabilities if there is no replacement.      [Solution](#)
  
5. There are 18 marbles in a bag: 6 red, 4 green, and 8 blue. We randomly pull three marbles. Find each of the following probabilities with replacement:
  - a) We pull 3 red marbles.
  - b) We pull 3 marbles of the same color.
  - c) We pull three marbles, all different colors.
  - d) e) f) Find the same probabilities if there is no replacement.      [Solution](#)
  
6. There are 16 red and 4 blue marbles in a bag. We randomly pull two marbles. If both are red, we win \$1. If both blue, we win \$4. Otherwise, we lose \$2. What is the expected value of this game for us, with and without replacement?      [Solution](#)
  
7. We roll a fair die. If we roll  $x$  and  $x$  is odd, we win the square of  $x$  in dollars. If  $x$  is even, we win the square of  $x$  in dollars. What is the expected value of this game for us?      [Solution](#)
  
8. There are 12 marbles in a bag. We randomly pull a marble. If it is red, we win \$8. If it is blue, we lose \$7. How many redmarbles are i the bag if the expected value of this game for us is  $-0.75$  dollars?      [Solution](#)
  
9. We roll two dice. If the numbers rolled are different, we pay 2 dollars. If we roll the same number, then we will win the sum of the number rolled (i.e if we roll a 5 and a 5, we win \$10). What is the expected value of this game for us?      [Solution](#)
  
10. Find the present value of the following three payments: \$1400 today, \$1400 a year from today, and \$1400 two years from today. Assume an annual compound interest rate of 6%, compounded monthly.      [Solution](#)

## 5 College Algebra

1. Solve  $\frac{5x - 4}{3x - 2} < \frac{7}{4}$

[Solution 1](#)    [Solution 2](#)

2. Simplify each of the given expressions.

a)  $\frac{2^{x+1} \cdot 5^{x-2}}{10^{x-2}}$     [Solution](#)

3. Find an equation for the tangent line drawn to the circle  $(x - 4)^2 + (y - 9)^2 = 40$  to the point  $P(10, 11)$ .    [Solution](#)

4. Find the domain of each of the given functions.

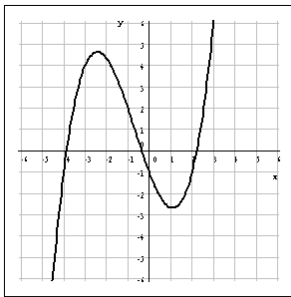
a)  $f(x) = \frac{1}{x^2 - 2x - 35}$

b)  $f(x) = \sqrt{x^2 - 2x - 35}$

c)  $f(x) = \ln(x^2 - 2x - 35)$     [Solution](#)

5. Find an equation for the set of all points that are twice as far from  $A(-3, -1)$  than from  $B(9, 5)$ .    [Solution](#)

6. Given the graph of  $y = f(x)$ . Graph  $g(x) = |f(x)|$  in the same coordinate system.    [Solution](#)



Graphing Factored Polynomials

[Part 1A](#),   [Part 1B](#),   [Part 1C](#)

[Graphing Factored Polynomials - Part 2](#)

Logarithms 2

[Part 0 - the Ingredients](#)    [Part 3 - Rules 5 and 6](#)

[Part 1 - Rule 3](#)    [Part 4 - Final Thoughts](#)

[Part 2 - Rule 4](#)

7. Suppose that  $A = \log_3 2$ . Express each of the following in terms of  $A$ .

- a)  $\log_3 18$     c)  $\log_3 \left(\frac{27}{4}\right)$     e)  $\log_{18} 24$   
 b)  $\log_3 24$     d)  $\log_2 3$     [Solution](#)

[Why  \$i^2\$  is  \$-1\$ ?](#)

[Summation Notation](#)

8. Compute each of the given sums. (Note: the solution uses  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$  after induction, and there is no mentioning of arithmetic sequences.)

a)  $\sum_{k=1}^{50} (8k - 3)$     c)  $\sum_{k=20}^{50} (8k - 3)$

b)  $\sum_{k=0}^{50} (8k - 3)$     [Solution](#)

9. Compute the given sums.

a)  $3^2 + 6^2 + 9^2 + \dots + 120^2$     [Solution](#)

b)  $0.8^2 + 1.6^2 + 2.4^2 + \dots + 120^2$     [Solution](#)

c)  $150^2 + 151^2 + \dots + 200^2$     [Solution](#)

d)  $1^2 + 3^2 + 5^2 + \dots + 201^2$     [Solution](#)

e)  $\sum_{k=5}^{200} (k^2 - 2k + 1)$     [Solution](#)

10. Find the slope of the tangent line drawn to the graph of  $y = -\frac{1}{2}x^2 + 3x + 1$  from the point  $P(4, 7)$ .    [Solution](#)

11. Find all values of  $p$  so that  $y = 4x + 2$  is tangent to  $y = px^2 + 8$ .    [Solution](#)

[Solving Quadratic Inequalities](#)

[How to take the Absolute Value of a Graph](#)

[Discontinuities of Rational Functions](#)   [Hole or vertical asymptote?](#)

[The Binomial Theorem](#)

[Why  \$0! = 1\$](#)

[Pascal's Triangle](#)

## 6 Trigonometry

### Arcs and Sectors in Circles

1. A sector in a circle is subtended by a central angle of  $40^\circ$ . The area of the sector is  $48 \text{ m}^2$ . Find the radius of the circle. [Solution](#)
2. Consider the triangle with sides 8, 16, and 17 units long. Is it a right triangle? [Solution](#)
3. Find the exact value of the distance between the points  $A(3, -6)$  and  $B(-3, 1)$ . [Solution](#)
4. Two sides of a right triangle are 9 and 40 units long. Find the exact value of the third side. [Solution](#)
5. Find the exact value of the height of the regular triangle with sides 14 units long. [Solution](#)
6. The base of a straight pyramid is a square with sides 14 units long. All other edges are 20 units long. Find the exact value of the height of the pyramid. [Solution](#)
7. The hypotenuse of a right triangle is 34 cm long. The difference between the other two sides is 14 cm. Find the sides of the triangle. [Solution](#)
8. The shortest side of a right triangle is 24 miles long. The difference between the other two sides is 4 miles. Find the missing sides. [Solution](#)
9. Find the perimeter and area of the parallelogram determined by the points  $A(-2, -3)$ ,  $B(21, -3)$ ,  $C(5, 9)$ , and  $D(-18, 9)$ . [Solution](#)

### Right Triangle Trigonometry

10. Find the exact value of all six trigonometric functions values of  $\beta$  in a right triangle in which  $a = 28 \text{ cm}$  and  $b = 45 \text{ cm}$ . [Solution](#)
11. Suppose that  $C$  is the center of a circle with radius 2 in and a point  $P$  is at a distance of 17in from  $C$ . Find an approximate value of the measure of the angle (in degrees) formed by the two tangent lines drawn to the circle from  $P$ . [Solution](#)

### Radian Measure of Angles

#### Unit Circle Trigonometry Definitions

- Part 1. [Scaling Down Right Triangles](#)
- Part 2. [Rotational Angles](#)
- Part 3. [Unit Circle Definition of Sine and Cosine](#)
- [All Together](#) (This is another lecture)

12. Find the exact value of  $\sin \alpha$  if we know that  $\tan \alpha = -2$ . [Solution](#)
13. Prove that the following is true for all triangles. If  $\alpha$ ,  $\beta$ , and  $\gamma$  are angles in the triangle, then  $\sin(\alpha + \beta) = \sin \gamma$ . [Solution](#)

14. Solve the given equations.

a)  $\sin \alpha + 1 = 2 \cos^2 \alpha$  [Solution](#)

b)  $\sin \gamma = \cos 2\gamma$  [Solution](#)

c)  $-\cos \alpha = \sin 2\alpha$  [Solution](#)

15. Find  $\sin \alpha$  if we know that

a)  $\tan \alpha = -2$

b)  $\tan \alpha = -2$  and  $\alpha$  is not in the 2nd quadrant. [Solution](#)

### Trigonometric Identities 2

#### Proving the Law of Cosines

#### Proving the Sum Formulas Using Vectors

#### Deriving All Other Compound Angle Formulas

16. Simplify the given expression.

$$\frac{\tan 32^\circ + \tan 28^\circ}{1 - \tan 32^\circ \tan 28^\circ} \quad \text{Solution}$$

17. Suppose that  $\alpha$  is an angle such that  $\sin \alpha = \frac{15}{17}$  and  $\alpha$  is not in the second quadrant. Compute the exact value of  $\sin 2\alpha$ . [Solution](#)

18. Suppose that  $\alpha$  is an angle such that  $\sin \alpha = -\frac{5}{13}$  and  $\alpha$  is not in the fourth quadrant. Compute the exact value of  $\cos 2\alpha$ . [Solution](#)

19. Suppose that  $\alpha$  is an angle such that  $\sin \alpha = -\frac{12}{13}$  and  $\alpha$  is not in the third quadrant. Compute the exact value of  $\tan 2\alpha$ . [Solution](#)

20. Find an equation for the line that bisects the angle formed between the line  $y = \frac{45}{28}x$  and the positive part of the  $x$ -axis. [Solution](#)

21. Prove each of the given identities.

a)  $\tan \alpha = \frac{\sin 2\alpha}{1 + \cos 2\alpha}$  [Solution](#)

b)  $\frac{\cos 2\alpha}{1 + \cos 2\alpha} = \frac{\tan \alpha}{\tan 2\alpha}$  [Solution](#)

c)  $\frac{1 + \cos 2\alpha}{\sin 2\alpha} = \cot \alpha$  [Solution](#)

### Sum-Product Identities

22. Suppose that  $C$  is the center of a circle with radius 9in and a point  $P$  is at a distance of 14in from  $C$ . Find exact value of sine of the angle that is formed by two tangent lines drawn to the circle. [Solution](#)

23. Suppose that one angle in a right triangle is  $24^\circ$ . Find all the sides if we also know that the perimeter of the triangle is 24 units. [Solution](#)



24. Suppose that  $\alpha$ ,  $\beta$ , and  $\gamma$  are the three angles in a triangle, opposite sides  $a$ ,  $b$ , and  $c$ , correspondingly. Prove that if  $\frac{\sin^2 \alpha}{\cos \alpha} = \frac{a^2}{bc}$ , then the triangle has a right angle. [Solution](#)

25. Compute the radii of the inscribed and superscribed circle for the triangle with sides 12, 16, and 20 units long. [Solution](#)

26. Compute the radii of the inscribed and superscribed circle for the triangle with sides 12, 20, and 20 units long. [Solution](#)

Proving The Area Formula  $A = \frac{1}{2}R^2 (\sin 2\alpha + \sin 2\beta + \sin 2\gamma)$

[Inverse Trigonometric Functions - Part 1](#), [Part 2](#)

## 7 Calculus

### Lectures

Average Speed and Velocity

Average Velocity

Complete Analysis of a Function - Part 1

Limits at Infinity - Part 1 and Part 2

Two-Sided Limits

The First Derivative Test

Proving the Intermediate Value Theorem

The Bolzano-Weierstrass Theorem

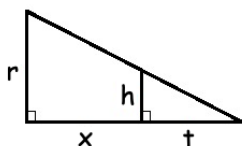
A Definition of  $e$

### Examples

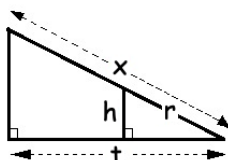
The problems here are not calculus problem, they are just part of the review at the beginning of the course.

1. Express  $x$  in terms of the other variables in the picture.

**Solution**

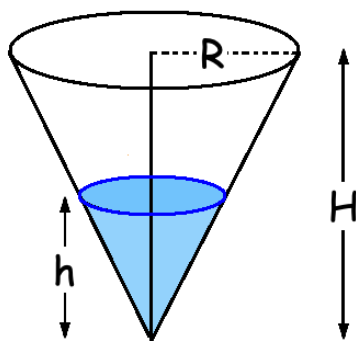


a)



b)

2. A water tank is of the shape of a cone . The tank is  $H = 20$ ft tall and the circular top edge of the cone has a radius  $R = 5$ ft long. If the surface of the water is at a height of  $h = 3$ ft, what is the surface area of the top of the water? **Solution**



3. Compute the given limits.

a)  $\lim_{x \rightarrow \infty} \frac{2^x + 2^{-x}}{2^x - 2^{-x}}$

4. Find all relative extrema for the function  $f(x) = 2x^3 + 6x^2 - 54x - 15$ . State the intervals over which  $f$  is increasing and decreasing. **Solution**

5. Find the slope of the tangent line drawn to the graph of  $y = -\frac{1}{2}x^2 + 3x + 1$  from the point  $P(4, 7)$ . **Solution**

6. Find all values of  $b$  and  $c$  so that  $y = 10x + 1$  is tangent to  $f(x) = x^3 + bx^2 + cx - 3$ . **Solution**

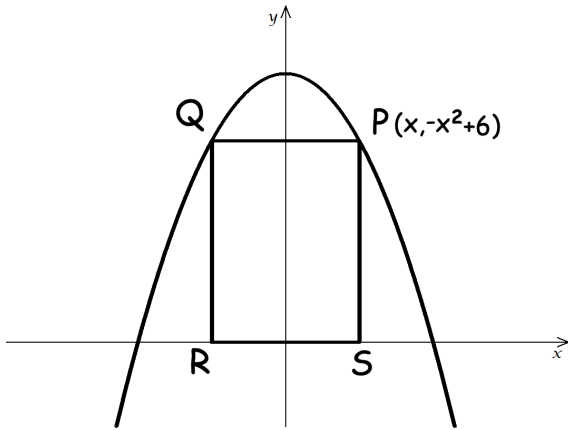
7. Find an equation of the common tangent lines drawn to the graphs of  $y = x^2$  and  $y = (x - 2)^2 + 12$ . **Solution**

8. Find all values of  $p$  so that  $y = 4x + 2$  is tangent to  $y = px^2 + 8$ . **Solution**

9. Find the greatest possible value of the product of two positive numbers if the sum of one number and the square of the other is 6. **Solution**

b)  $\lim_{x \rightarrow -\infty} \frac{2^x + 2^{-x}}{2^x - 2^{-x}}$  **Solution**

10. Let  $P(x, y)$  be a point on the graph of  $y = -x^2 + 6$  with  $0 < x < \sqrt{6}$ . Let PQRS be a rectangle with one side on the x-axis and two vertices on the graph, as shown on the picture. Find the exact value of the greatest possible area of such a rectangle. [Solution](#)



11. Find the point(s) on the graph of  $y = 5x^2$  that is closest to the point  $P(0, 2)$ . [Solution](#)

12. Which square-based open box with a surface of 2400 square inches has the greatest volume? [Solution](#)

13. Compute the given limit. You may assume that  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$  and  $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x \quad \text{Solution}$$

14. Compute each of the given indefinite integrals.

a)  $\int e^{3x-2} dx$

e)  $\int \cos^2 x dx$

b)  $\int (5x + 8)^{10} dx$

f)  $\int \frac{1}{3-x} dx$

c)  $\int \cos(2x - \pi) dx$

g)  $\int \frac{6x-1}{2x+5} dx$

d)  $\int \sin x \cos x dx$

[Solution](#)

## 8 Geometry

Introduction to Angles

The Triangle Inequality

Similar Triangles

Introduction to Construction

Tangents to Circles

The Perpendicular Bisector and the Superscribed Circle

The Angle Bisector and the Inscribed Circle

Thales's Theorem

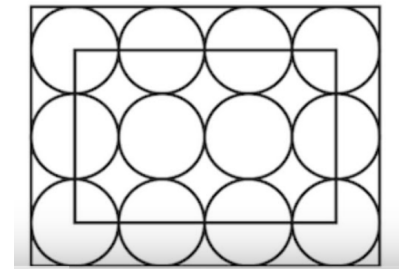
Proving The Angle Bisector Theorem

Proving The Area Formula

$$A = \frac{1}{2}R^2 (\sin 2\alpha + \sin 2\beta + \sin 2\gamma)$$

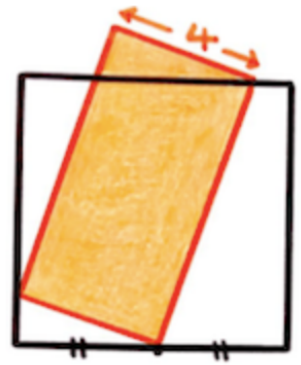
Inverse Trigonometric Functions - Part 1, Part 2

- The picture shows twelve congruent (identical) circles. The rectangle drawn around the circles (so all four sides are tangent lines) has a perimeter of 168 cm. The smaller rectangle passes through the centers of ten circles. What is the perimeter of this smaller rectangle? [Solution](#)



- Compute the exact value of the area of a triangle with sides 18, 11, and 11 units long. [Solution](#)

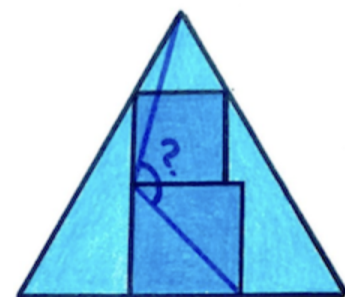
- An orange rectangle and a black square. What's the shaded area? (Note that the tick-marks on the bottom side indicate that the two line segments are of the same length.) [Solution](#)



- Find the exact value of the height of the regular triangle with sides 14 units long. [Solution](#)

- Our flagpole was originally 50 feet tall. A storm broke it into two pieces last night. The bottom piece is attached to the ground and it is now  $x$  feet tall. The top part is still attached to the bottom part, but the tip of the flagpole is now on the ground, at 40 feet away from its base. Find the exact value of  $x$ . [Solution](#)

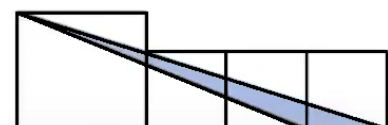
- The picture shows two squares written in an equilateral (regular) triangle. What is the measure of the marked angle? [Solution](#)



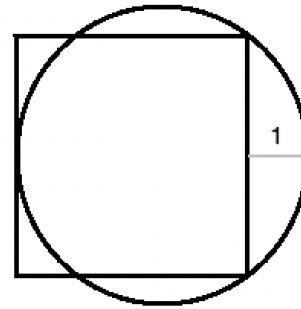
- We drew the height belonging to the hypotenuse of a right triangle. The height split the hypotenuse into parts 18 and 50 units long. Find the exact value of the shorter side and the height drawn to the hypotenuse. [Solution](#)

- An arch is in the shape of a semicircle. At a point along the base 3 feet from an end of the arch, the height of the arch is 5 feet. Find the maximum height of the arch. [Solution](#)

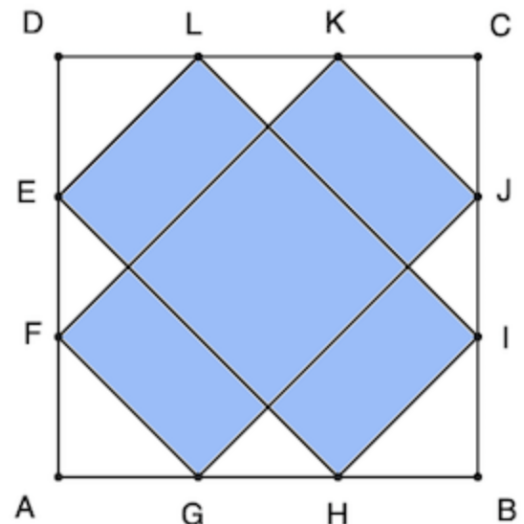
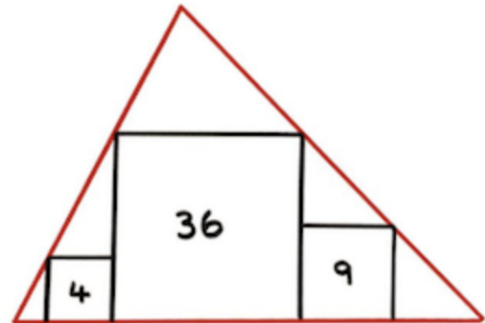
- The picture shows four squares. The sides of the smaller squares are 12 units long. How long are the sides of the larger square? [Solution](#)



10. What is the area of the square shown on the picture? [Solution](#)
11. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 4 units. What is the length of each side of the octagon? [Solution](#)
12. Find the perimeter and area of the regular 8-sided polygon (also called octagon) if the distance between its opposite sides is 9 units. [Solution](#)
13. The areas of the three squares are given. What's the area of the red triangle? [Solution](#)

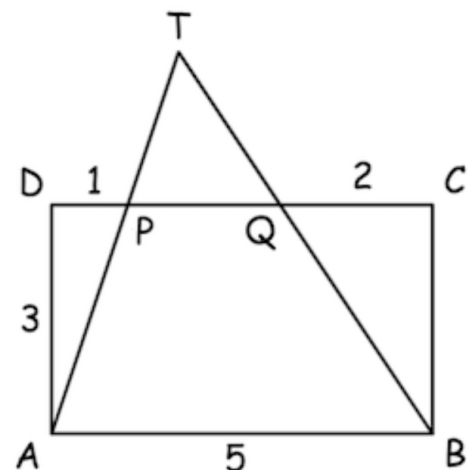


14. The square shown on the picture has area 225 unit<sup>2</sup>. Line segments  $AG, GH, HB, BI, IJ, JC, CK, KL, LD, DE, EF,$  and  $FA$  are all equally long. Find the exact value of the shaded region. [Solution](#)
15. Suppose that  $ABCD$  is a rectangle. The midpoints of the longer sides,  $AD$  and  $BC$  are connected, cutting the rectangle into two rectangles, both similar to the original triangle. If  $\overline{AB} = 2$ , then how long is side  $BC$ ? [Solution](#)

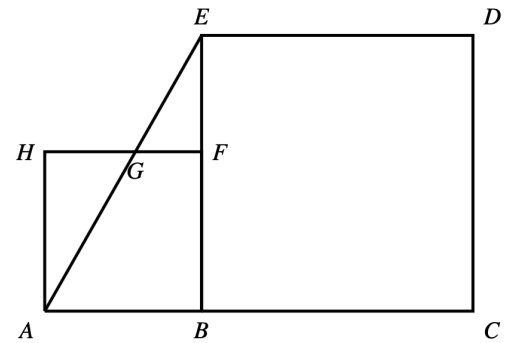


16. Consider rectangle  $ABCD$  with sides 3 and 5 units long as shown on the picture.  $T$  is a point outside the rectangle,  $P$  is the intersection of  $AT$  and  $CD$ ,  $Q$  is the intersection of  $BT$  and  $CD$  such that  $DP = 1$  and  $CQ = 2$ . Find the exact value of the area of triangle  $PQT$ .

[Solution](#)

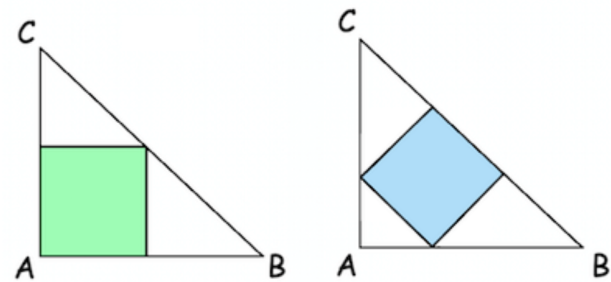


17. The picture shows two squares. The smaller one,  $ABFH$  has sides 1 unit long. We further know that angle  $EAB = 60^\circ$ . Find the exact value of the length of each of the given line segments.



$AE$   $BE$   $GH$   $AG$   $FG$   $EG$   $EF$  [Solution](#)

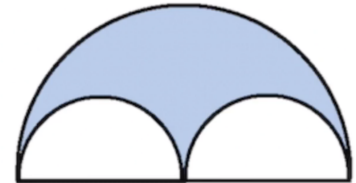
18. We drew a square into the same isosceles right triangle in two different ways as shown on the picture.



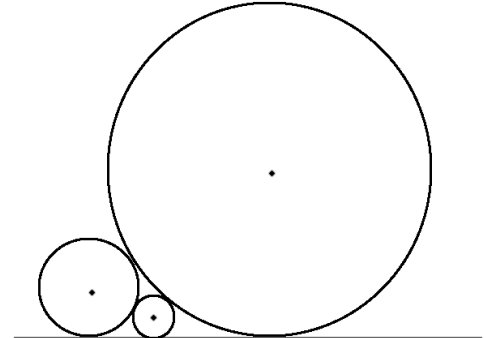
- a) Find the length of side  $AB$  if the green square's area is  $81 \text{ ft}^2$ .  
 b) Find the area of the blue square.

[Solution](#)

19. The picture shows three semi-circles. The large semi-circle has radius 6 units. Find the exact value of the area of the shaded region. [Solution](#)

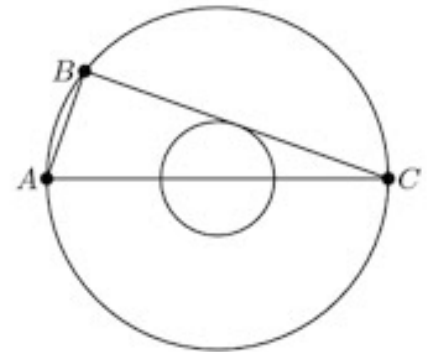


20. The circles on the picture shown are tangent to each other. Find the radius of the smallest circle if the radii of the other circles are 1 and 4 units long.



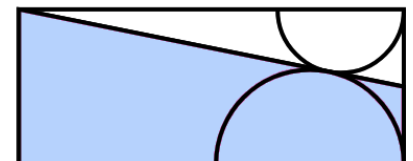
[Solution](#)

21. An isosceles trapezoid is written into a circle. The parallel sides of the trapezoid are 14 and 16 units long. Its height (the line segment connecting the parallel sides, and perpendicular to both) is 5 units long. Find the exact value of the radius of the circle. [Solution](#)



22. The two circles shown on the picture have the same center and their radii are in a ratio of 3 to 1. In triangle  $ABC$ , side  $AC$  is a diameter in the larger circle and side  $BC$  is tangent to the smaller circle. Side  $AB$  is 18 units long. Find the exact value of the length of side  $AC$  [Solution](#)

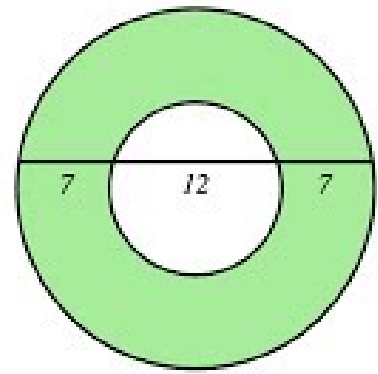
23. The sides of the rectangle are 6 and 20 units long. Find the area of the shaded region. [Solution](#)



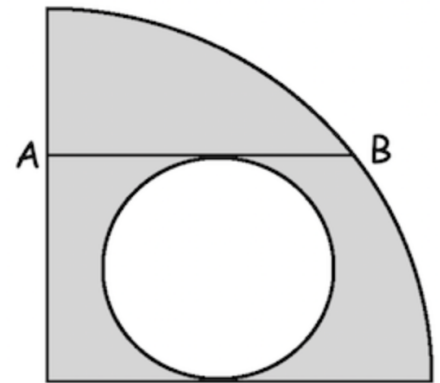
24. Find the area of the shaded region. [Solution](#)

25. Given a circle and a point  $P$  outside of the circle, construct the tangent lines that can be drawn to the circle from point  $P$ .

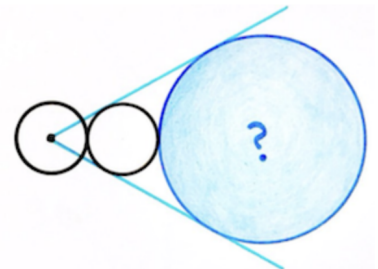
[Solution](#)



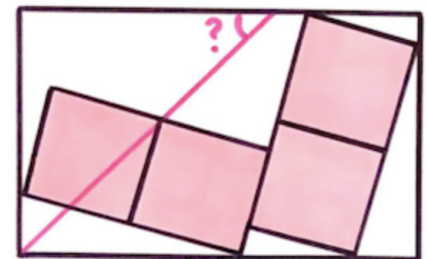
26. A circle in a quarter circle. Find the exact value of the shaded area if line segment  $AB$  is 6 units long. [Solution](#)



27. The small circles each have area  $5 \text{ unit}^2$ . What is the area of the large circle? [Solution](#)



28. Four squares in a rectangle. Find the measure of the marked angle. [Solution](#)



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