Calculus

Lectures

Average Speed and Velocity
Average Velocity

Complete Analysis of a Function - Part 1

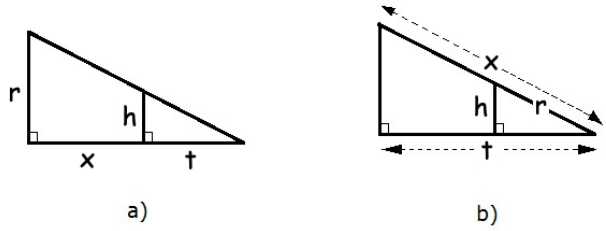
Limits at Infinity - Part 1 and Part 2

Two-Sided Limits
The First Derivative Test
Proving the Intermediate Value Theorem
The Bolzano-Weierstrass Theorem
A Definition of e

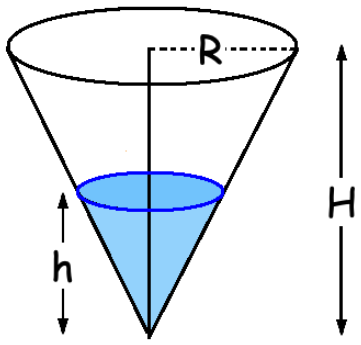
Examples

The problems here are not calculus problem, they are just part of the review at the beginning of the course.

1. Express x in terms of the other variables in the picture. Solution



2. A water tank is of the shape of a cone . The tank is $H = 20\text{ft}$ tall and the circular top edge of the cone has a radius $R = 5\text{ft}$ long. If the surface of the water is at a height of $h = 3\text{ft}$, what is the surface area of the top of the water? Solution



3. Compute the given limits.
   a) $\lim\limits_{x\to\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$
   b) $\lim\limits_{x\to-\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$ Solution

4. Find all relative extrema for the function $f(x) = 2x^3 + 6x^2 - 54x - 15$. State the intervals over which f is increasing and decreasing. Solution

5. Find the slope of the tangent line drawn to the graph of $y = -\dfrac{1}{2}x^2 + 3x + 1$ from the point $P(4, 7)$. Solution

6. Find all values of b and c so that $y = 10x+1$ is tangent to $f(x) = x^3 + bx^2 + cx - 3$. Solution

7. Find an equation of the common tangent lines drawn to the graphs of $y = x^2$ and $y = (x-2)^2 + 12$. Solution

8. Find all values of p so that $y = 4x + 2$ is tangent to $y = px^2 + 8$. Solution

9. Find the greatest possible value of the product of two positive numbers if the sum of one number and the square of the other is 6. Solution

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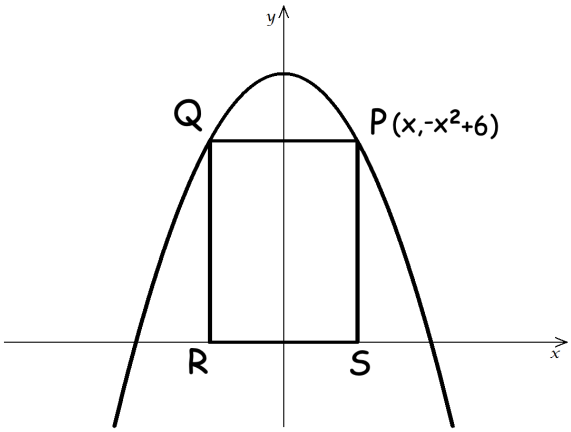
10. Let $P(x, y)$ be a point on the graph of $y = -x^2 + 6$ with $0 < x < \sqrt{6}$. Let PQRS be a rectangle with one side on the xaxis and two vertices on the graph, as shown on the picture. Find the exact value of the greatest possible area of such a rectangle. Solution



11. Find the point(s) on the graph of $y = 5x^2$ that is closest to the point $P(0, 2)$. Solution

12. Which square-based open box with a surface of 2400 square inches has the greatest volume? Solution

13. Compute the given limit. You may assume that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$ and $\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x = e$

$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x$ Solution

14. Compute each of the given indefinite integrals.

a) $\int e^{3x-2}dx$

b) $\int (5x+8)^{10}\,dx$

c) $\int \cos(2x-\pi)\,dx$

d) $\int \sin x \cos x dx$

e) $\int \cos^2 x dx$

f) $\int \frac{1}{3-x}dx$

g) $\int \frac{6x-1}{2x+5}dx$

Solution

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