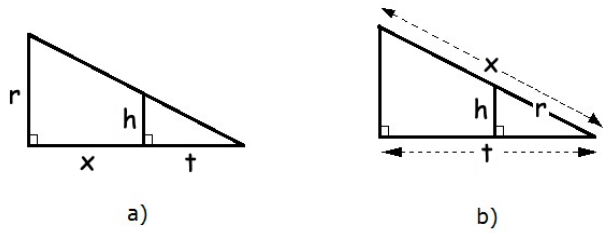
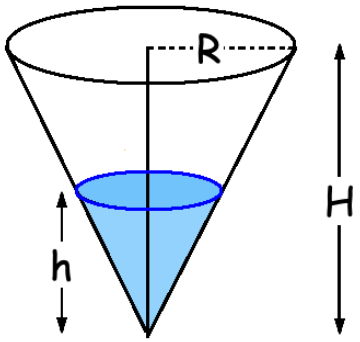
Calculus

1. Express x in terms of the other variables in the picture. Solution



2. A water tank is of the shape of a cone . The tank is $H = 20\text{ft}$ tall and the circular top edge of the cone has a radius $R = 5\text{ft}$ long. If the surface of the water is at a height of $h = 3\text{ft}$, what is the surface area of the top of the water? Solution



3. Compute the given limits.

   a) $\lim_{x\to\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$

   b) $\lim_{x\to-\infty} \dfrac{2^x + 2^{-x}}{2^x - 2^{-x}}$ Solution

4. Find all relative extrema for the function $f(x) = 2x^3 + 6x^2 - 54x - 15$. State the intervals over which f is increasing and decreasing. Solution

5. Find the slope of the tangent line drawn to the graph of $y = -\dfrac{1}{2}x^2 + 3x + 1$ from the point $P(4, 7)$. Solution

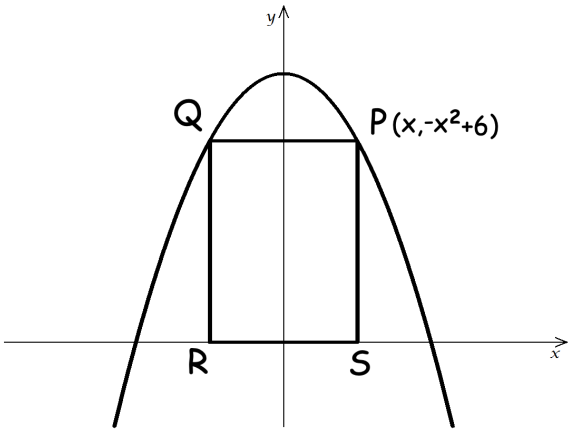
6. Find all values of b and c so that $y = 10x+1$ is tangent to $f(x) = x^3 + bx^2 + cx - 3$. Solution

7. Find an equation of the common tangent lines drawn to the graphs of $y = x^2$ and $y = (x-2)^2 + 12$. Solution

8. Find all values of p so that $y = 4x + 2$ is tangent to $y = px^2 + 8$. Solution

9. Find the greatest possible value of the product of two positive numbers if the sum of one number and the square of the other is 6. Solution

10. Let $P(x, y)$ be a point on the graph of $y = -x^2 + 6$ with $0 < x < \sqrt{6}$. Let PQRS be a rectangle with one side on the xaxis and two vertices on the graph, as shown on the picture. Find the exact value of the greatest possible area of such a rectangle. Solution



11. Find the point(s) on the graph of $y = 5x^2$ that is closest to the point $P(0, 2)$. Solution

12. Which square-based open box with a surface of 2400 square inches has the greatest volume? Solution

13. Compute the given limit. You may assume that $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = e$ and $\lim_{x\to-\infty}\left(1+\dfrac{1}{x}\right)^x = e$

    $\lim_{x\to\infty}\left(1+\dfrac{3}{x}\right)^x$ Solution

14. Compute each of the given indefinite integrals.

    a) $\int e^{3x-2}dx$

    b) $\int (5x+8)^{10}\,dx$

    c) $\int \cos(2x-\pi)\,dx$

    d) $\int \sin x \cos x dx$ Solution

    e) $\int \cos^2 x dx$

    f) $\int \dfrac{1}{3-x}dx$

    g) $\int \dfrac{6x-1}{2x+5}dx$

Lecture Videos

Average Speed and Velocity
Average Velocity

Complete Analysis of a Function - Part 1

Limits at Infinity - Part 1 and Part 2

Two-Sided Limits
The First Derivative Test
Proving the Intermediate Value Theorem
The Bolzano-Weierstrass Theorem
A Definition of e

